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The Effects of a Geometry Intervention on Geometry Skills for Elementary Students with Learning Disabilities

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The Effects of a Geometry Intervention on Geometry Skills for Elementary Students with Learning Disabilities

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Dedication

To my mom, Xiujuan Lyu, who always thinks for me.

To my love, Lukas Gradl, who supports me during the tough time and the big moments.

To future me, who should be brave to face new challenges.

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Abstract

The Effects of a Geometry Intervention on Geometry Skills for Elementary Students with Learning Disabilities

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Geometry education is a core content area for Kindergarten to 12th grade (K-12) mathematics education in the U.S. Success in geometry can benefit students in many aspects; for example, in pursuit of higher education or jobs related to science, technology, engineering and mathematics (STEM). However, students with learning disabilities (LD) usually face challenges in solving geometry problems. There are limited empirical studies on geometry interventions for students with LD. Of those that have been conducted, few have focused on geometry for students with LD at the lower grades. Recognizing this gap in the literature, the purpose of this study was to examine the effect of a geometry intervention on the geometry performances of fourth-graders with LD using a multiple baseline design. The research questions that guided this study were as follows: (a) What is the immediate effect of a geometry intervention on the geometry performances of fourth- and fifth-grade students with LD as measured by a proximal measure (adapted easyCBM)? (b) To what extent do the fourth- and fifth-grade students with LD maintain

their geometry performance one week after the conclusion of the intervention as measured by a proximal measure (adapted easyCBM)? (c) To what extent do the fourth- and fifth-grade students with LD generalize their geometry knowledge to a distal measure (KeyMath-3 geometry subtest)? (d) What are the perspectives of the fourth- and fifth-grade students with LD on the geometry intervention?

The intervention included empirically validated instructional components (ICs), such as guided practice, and the use of multiple representations (Swanson & Sachse-Lee, 2000). The interventionist implemented seven lessons with effective ICs on the grade-aligned geometry concepts and skills based on the Common Core State Standards for Mathematics (CCSSM) standards, including identifying of shapes, learning of the shape attributes, and solving the perimeter and area of various polygons, and understanding the concept of symmetry lines. The measures used for the present study included the adapted easyCBM geometry measures and the KeyMath-3 geometry subtest. A social validity measure was administered to capture students' perspectives on the geometry intervention. The limitations, future research directions, and implications for practice in teaching geometry were also discussed.

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Chapter 1: Introduction

CHALLENGES OF GEOMETRY EDUCATION

Mathematics provides a powerful tool for individual learners to explore and understand the physical world. Geometry, an area of mathematics, offers learners a system to describe and make sense of the space around them (National Research Council, 2009). Geometry is an essential component of the elementary mathematics curriculum. It also gives students the necessary experience in solving problems by applying their knowledge of shapes and shape properties (Musser & Burger, 1994). The past presidents of the National Council of Teachers of Mathematics have emphasized the importance of teaching geometry to students from kindergarten to the 12th grade (K-12) (Kepner, 2009; Shaughnessy, 2011). Despite that, the states and national organizations often prioritize other mathematical areas in curriculum development or assessment, such as arithmetic operations, or algebraic concepts and procedures (Shaughnessy, 2011).

According to the findings of the 2015 Trends in International Mathematics and Science Study (TIMSS; Mullis et al., 2016), the poor geometry performance of the students across all grades represents a growing problem in the US. The results of the TIMSS assessment enable a comparison between US students and students in other countries with respect to mathematics and the sciences. The assessment includes a composite mathematics score and an individual geometry score. In 2015, US fourth-graders ranked 14th out of 49 countries for the composite score and 23rd for the geometry score. Similarly, US eighth-graders ranked 10th out of 39 countries for the composite score and 15th for the geometry score.

Over the last decade, the findings of the US National Assessment of Educational Progress (NAEP; National Center for Education Statistics, 2019) also reveal another achievement gap in geometry, which is between students with and without disabilities. Students identified as having disabilities include those enrolled in an individualized education program (IEP) or those whose rights are protected under Section 504 of the Rehabilitation Act of 1973. Taking the achievement gaps of fourth-graders as an example, the achievement discrepancy in the geometry domain between students with and without disabilities widened from 14 points in 2009 to 31 points in 2019 (see Table 1.1).

Table 1.1

Average Scale Score for Grade 4 Geometry Scale of NAEP 2009–2019

Year	Student without disabilities	Students with disabilities
2019	245	214
2017	237	215
2015	238	221
2013	243	224
2011	243	224
2009	240	226

A low geometry achievement could also influence the scant number of students pursuing occupations related to science, technology, engineering, and mathematics (STEM), as geometry skills and mathematical-reasoning abilities are found to be highly associated with the expectations and qualifications for STEM-related jobs (Carnevale et al., 2011). Analysis conducted by the Occupational Information Network at the Center on Education and Workforce at Georgetown University identifies an unexpected shortage of STEM workers in the US (Carnevale et al., 2011). Given the persistently high wages of STEM and STEM-related jobs, it is also surprising that large numbers of people in the US with STEM skills diverted from a STEM career while in school or in the early phase of their career (Carnevale et al., 2011). Improving the quality of geometry instruction may help solve the problem of the scarcity of STEM workers in the US by preparing students through giving them the necessary mathematical skills, abilities, confidence, and interest to pursue STEM jobs (Carnevale et al., 2011).

GEOMETRY AND OTHER MATHEMATICS DOMAINS

Geometry is highly important to other areas of mathematics. Spatial sense, measurement, and algebraic skills are required in solving many math problems. According to the National Council of Teachers of Mathematics (NCTM, 2000), geometry can help students represent and solve complex problems involving fractions, histograms, or coordinate planes. The National Mathematics Advisory Panel (NMAP) (2008) also argues that knowledge of geometry concepts is critical for the study of algebra. These crucial concepts include the knowledge and ability to analyze the properties of two- and three-dimensional (2D and 3D) shapes, and to determine their perimeter, area, volume, and surface (NMAP, 2008). The knowledge of elementary geometry also is also related to secondary mathematics content, such as trigonometry or statistics (Clements & Battista, 1992; Fabiyi, 2017; Hadi & Faradillah, 2020). Therefore, integrating geometry into other content areas in the mathematics curriculum provides students with extra opportunities and tools to explore and understand mathematical problems (Lappan, 1999).

Spatial Sense

Spatial sense supports geometry (Uttal & Cohen, 2012). Geometry and spatial sense are fundamental skills for mathematics education for PreK through the 12th grade (P-12) (Freudenthal, 1978; National Research Council, 2009; Wheatley & Reynolds, 1999). Spatial reasoning or spatial sense is vital for spatial thinking, which involves the mental representation and knowledge of shape relationships. Spatial sense is crucial for understanding patterns in art, nature, and architecture. In mathematics, geometry involves spatial-thinking and mental-representation skills.

Researchers have found that experts perceive and organize spatial knowledge around their abstract semantic knowledge; for example, by looking at a shape, one can immediately activate knowledge or a theorem related to the shape for a proof or other higher-order task (Koedinger & Anderson, 1990). Because better mental-representation skills can assist in geometry problem-solving, researchers have tried to provide students with multiple visual representations of geometry problems. For example, Zhang et al. (2014) conducted research on test accommodations of geometry problems and found that students with geometry difficulties performed better when presented with visual representations on geometry tests.

Measurement

Measurement connects geometry with number sense. It is a real-world application of mathematics (Gravemeijer et al., 2016; National Research Council, 2009). According to

NCTM standards, P-12 students should understand the measurable attributes of objects, the use of standard units for measurement, and the application of appropriate techniques to make comparisons and estimations. By acquiring fundamental knowledge of measurement and geometry, students are able to solve problems about the area, perimeter, surface, volume, and other aspects of shapes (van de Walle, 2004).

Algebra

Algebra also has a close relationship to geometry. The connections between algebra and geometry are established through elementary geometry. A report from NMAP (2008) indicated that the knowledge of fractions, and particular aspects of measurement and geometry are foundational to algebra. Geometric shapes help students to conceptualize fractions and ratios (National Research Council, 2009). These shapes also help with learning coordinate planes so that students can describe a location in space or analyze geometry problems using pairs of numbers on a coordinate plane (National Research Council, 2009), which is an important step toward abstract thinking.

STUDENTS WITH LEARNING DISABILITIES AND LEARNING GEOMETRY

Students with learning disabilities (LD) are a heterogeneous group with one shared characteristic: Their ability to learn and benefit from general education is greatly hindered by their disabilities (Individuals with Disabilities Education Improvement Act, 2017). Data from the U.S. Department of Education indicated that 34.4% of the students who were eligible for special education services in the U.S. are students with LD. Many students with LD experience a range of problems when learning and applying mathematics to other situations (Gartland & Strosnider, 2018). They may also encounter issues in the performances of mathematics procedures or cognitive skills when solving mathematics problems, such as working memory (Geary, 2004). Moreover, when instruction relies heavily upon abstract definitions or terms, language deficits may also hinder learning mathematics (Bley & Thornton, 2001; Ives, 2007).

The Common Core State Standards Initiative (CCSS, 2010) set new standards for teaching geometry by emphasizing conceptual understanding and procedural skills. For students with LD and limited instructional support, the acquisition of conceptual and procedural knowledge can be a demanding task because of their deficits (Satsangi, Hammer, & Bouck, 2019). Many students with LD receive instruction in general education classrooms and are evaluated using the same assessment tools as their peers. Therefore, geometry interventions for students with LD should be put in place to improve their geometry outcomes.

National and state-level mathematics standards are designed to address the essential concepts and skills that students need to learn. At the national level, the Common Core State Standards for Mathematics (CCSSM, 2010) directs the educators to ensure that U.S. students receive the mathematics education needed at each grade level. The standards apply to most struggling learners, including those with LD. In 2013, the CCSSM writing team (2013) published the work of Progressions for the CCSSM, which is an additional resource for mathematics teachers and researchers. This resource was written with the input of mathematicians and educational researchers, and it provided additional explanations for the connections between general standards and geometry requirements. According to the

CCSSM standards of geometry, kindergarteners should be able to understand shapes; first- through third- graders should be able to reason about the shapes and their attributes; fourth- through eighth- graders should be able to discriminate between shapes, analyze shapes with the help of lines, angles, coordinate planes, or physical models, and calculate area and volume; and high schoolers should have formalized geometric knowledge and view geometry through a careful and systematic perspective.

Geometry is also embedded in statewide standards and assessments. For example, in Texas, the Texas Essential Knowledge and Skills (TEKS, 2012), provides detailed requirements about what students should know and be able to do at different grade levels from PreK to Grade 12. TEKS standards are given for mathematics instruction in elementary school, middle school, and high school. This includes advanced courses, such as statistics and calculus. From PreK through the fifth grade, the TEKS requires students to be able to identify and analyze the attributes of geometric figures, understand transformational geometry, and know how to use coordinate planes to graph and solve problems at the elementary level.

STATEMENT OF THE PROBLEM

The purpose of this study is to examine the effects of geometry interventions on the performance of elementary students with LD. Geometry is an important mathematics content area that is tested by national standards (e.g., NAEP, 2017) for K-12 students in the U.S. Nonetheless, U.S. students have scored poorly on basic geometric concepts and problem-solving skills at the elementary and middle school level (Clements & Battista, 1992; NAEP, 2019; TIMSS, 2015). To better prepare students, researchers suggest using high-quality mathematics instruction for struggling learners, especially students with LD (Gersten et al., 2009; Swanson & Sachse-Lee, 2000). In addition, the data indicates that teachers may need to address students' misconceptions of geometry in the early grades (Clements & Battistia, 1992). For example, some students have problems with shapes in different orientations. They may not identify a shape as a square when its base is not horizontal. The following outlines the research questions the investigator will pose.

RESEARCH QUESTIONS

To address the aforementioned problem, the investigator will pursue the following research questions:

1. What is the immediate effect of a geometry intervention on the geometry performances of fourth- and fifth-grade students with LD as measured by a proximal measure (adapted easyCBM)?
2. To what extent do the fourth- and fifth-grade students with LD maintain their geometry performance one week after the conclusion of the intervention as measured by a proximal measure (adapted easyCBM)?
3. To what extent do the fourth- and fifth-grade students with LD generalize their geometry knowledge to a distal measure (KeyMath-3 geometry subtest)?
4. What are the perspectives of the fourth- and fifth-grade students with LD on the geometry intervention?

Chapter 2: Literature Review

Recently, increasing attention has been paid to the role of geometry in general education (Forsythe, 2007; Weber, 2003). Nonetheless, the research results suggest that there is a critical shortage of geometry interventions for students with special needs (Bergstrom & Zhang, 2016), especially for students with LD (Liu et al., in press). In this chapter, the pertinent literature on research-based geometry interventions in special education is reviewed. Particular consideration is given to studies that examine the potential benefits of such interventions for students with Learning Disabilities (LD).

REVIEW OF GEOMETRY INTERVENTIONS

Geometry Instruction for Students with Disabilities

Research about teaching geometry to students with disabilities is particularly scarce. Bergstrom and Zhang (2016) conducted a systematic review of studies that concerned teaching geometry to all students, including students with disabilities. Out of the thirty-two studies included in their study, only nine (28%) focused on teaching geometry to students with disabilities. The rest of the studies targeted either typical or gifted students. Nonetheless, eight out of the nine studies focusing on students with disabilities incorporated instructional strategies in their geometry interventions. For example, one study used a type of educational technology, which was a strategy known as virtual manipulatives, to teach geometry (Satsangi & Bouck, 2015). Instructional strategies refer to the way teachers present curricula, engage with students, and teach concepts and skills (Bergstrom & Zhang, 2016). In Bergstrom and Zhang, the identified instructional strategies were: (a) concrete-representational-abstract instructional sequence (Cass et al., 2003; Strickland & Maccini, 2012), (b) a combination of concrete-representational-abstract instructional sequences and model-based problem-solving instruction (Hord & Xin, 2015), (c) conceptual model-based problem-solving instruction (Xin & Hord, 2013), (d) lecture-based instructional techniques (Worry, 2011), (e) test accommodations (Kang & Zentall, 2011; Zhang et al., 2012; Zhang et al., 2014), and (f) the use of technology (Satsangi & Bouck, 2015).

Researchers have conducted geometry interventions targeting four types of disabilities: intellectual disabilities (Browder et al., 2012; Creech-Galloway et al., 2013; Heinrich et al., 2016; Hord & Xin, 2015), Attention Deficit Hyperactivity Disorder (ADHD; Kang & Zentall, 2011), Autism (e.g., Dixon et al., 2016), and learning disabilities (e.g., Cihak & Bowlin, 2009).

Students with Intellectual Disabilities

The researchers of four studies that examined effective geometry instructions for students with mild intellectual disabilities all included middle school and high school students and used single case designs (i.e., multiple probe or multiple baseline designs). The geometry topics taught in these studies included skills that met school, state, or national mathematics standards (Browder et al., 2012; Heinrich et al., 2016), knowledge of the Pythagorean Theorem (Creech-Galloway et al., 2013), and area and volume problem-solving skills (Hord & Xin, 2015). Browder et al.'s (2012) multiple probe study used graphic organizer and adapted stories in their intervention to teach middle school students with mild intellectual disabilities. The researchers printed the materials in color and incorporated mathematics manipulatives to teach the lessons. At the end of the intervention, all participants demonstrated evidence of geometric skills acquisition. Therefore, the use of visual representation in combination with the graphic organizer improved the standard-aligned geometry skills of the students with disabilities.

In two other studies, the researchers employed an instructional method called simultaneous prompting procedures to teach secondary students with mild intellectual disabilities (Creech-Galloway et al., 2013; Heinrich et al., 2016). During the instructional phases, teachers used strategies such as task directions, problem-solving prompts (physical and verbal), responses, and timely feedback (praises). Creech-Galloway et al. (2013) used iPads (tablet computers), to present geometry problems in real-world contexts (e.g., a seamstress's shop), whereas Heinrich et al. (2016) used paper cards with printed geometry problems. The results of both studies indicated the positive effects of simultaneous prompting procedures when teaching the required geometry skills. The findings of both studies confirmed that students with mild intellectual disabilities can learn complex mathematics knowledge (i.e., geometry) with additional instruction.

Besides using prompting strategies, Hord and Xin (2015) examined the effects of a combination of concrete, semi-concrete, and abstract instructional sequencing and model-based problem-solving instructions to teach middle school students with mild intellectual disabilities. Their intervention focused on teaching problem-solving strategies for calculating area and volume. While teaching, Hord and Xin provided students multiple opportunities to manipulate concrete items (wooden blocks and figures), semi-concrete items (drawings of figures), and abstract items (formulas). Researchers also incorporated a mathematical model (an equation model of shape area and volume) to facilitate students transition from concrete to abstract models. The results of the study indicated the positive effects of this intervention model.

Students with Attention Deficit Hyperactivity Disorder

Only one study focused on teaching geometry concepts and skills to students with autism. Kang and Zentall (2011) conducted an experimental group design to identify the effects of the visual cues in a computer-based instructional program. The second- and fourth-graders with Attention Deficit Hyperactivity Disorder (ADHD), were randomly assigned to the treatment (images with high visual information) or control (Images with low Visual information) groups. The students in the treatment group had access to geometry problems accompanied by 3D images and visual aids (e.g., a light source and shadows). In the control group, the participants only saw the images without any visual aids. At the end of the study, the results indicated participants in the treatment group performed better than those in the control group (F (1, 11) = 12.59, p = 0.005). Using visual aids during the instruction period helped students with ADHD solve geometry problems.

Students with Autism

Researchers have also utilized equivalence-based instruction (EBI) to teach geometry concepts to students with autism. Dixon et al. (2016) conducted an intervention with stimulus-equivalence procedures. They used cards with pictures of geometric shapes on one side, written numbers that indicated the number of the sides of the shape on the other side, and gave the shape names vocally. The intervention incorporated positive reinforcement components and prompting procedures. For example, during the training sessions, the instructor selected a number card and accompanied this with the vocal stimulus, “How many sides does a pentagon have?” When a student answered correctly, the instructor would praise the student by saying, “Great job.” After the intervention was complete, all secondary-level participants with autism had successfully established knowledge of the relationships between each shape’s name, the number of sides of that shape, and that shape’s image. Although more research is needed to generalize these findings, the results of the study indicate that there is potential for students with autism to learn geometry.

In summary, there is limited research on how geometry concepts and skills are taught to students with intellectual disabilities, ADHD, and autism (Bergstrom & Zhang, 2016). However, the limited literature also reveals that these students benefit from additional geometry instruction and techniques such as multiple presentations, simultaneous promptings, visual cues, and opportunities for students to practice. Future researchers are encouraged to conduct more studies to examine the generalizability of these interventions to larger student populations, including for students with learning disabilities.

Students with Learning Disabilities

To understand the geometry improvements for students with LD, researchers need to find direct evidence to show the geometry outcomes of students with LD before and after the intervention. There is a research gap in this area. To extend existing literature, Liu et al. (in press) conducted a synthesis of geometry interventions that included students with LD and their data was disaggregated from other participants, such as that of students with other disabilities. Specifically, this study provided a summary of studies on the following: (a) geometry topics; (b) geometry intervention type (e.g., the use of technology); c) ICs (e.g., skills modeling); (d) the effectiveness of the geometry studies; and (e) the methodological rigor of the studies (i.e., the quality of the research).

Nine studies have been located that contain data for students with LD that could identify the changes of geometry outcomes. These studies included a total of 71 students with LD. Eight of the nine studies involved participants with LD from secondary schools (n = 69, 97%).

The nine studies only covered angle recognition, perimeter, area, and volume problems—neglecting the majority of the geometry concepts and skills listed in the national standards (i.e., CCSSM). Furthermore, all but two of the studies (Satsangi, Hammer, & Bouck, 2019; Xin & Hord, 2013) focused on geometry concepts and skills beneath the current level of the participants. A total of 97% of participants with LD were middle or high school students. However, the geometry skills taught in their interventions were based on elementary curriculums and did not include the standards set by the CCSSM. For example, one study included high schoolers whose geometry intervention involved elementary-level geometry topics (e.g., 2D perimeter problems; Cihak & Bowlin, 2009).

Liu et al. found that the researchers tended to use instructional strategies and technology in their geometry interventions for students with LD. Out of nine geometry interventions, four studies incorporated instructional strategies. Using modeling, prompting, guided practices, and independent practice, the researchers instructed secondary participants with LD to solve perimeter and area problems (Cass et al., 2003; Kozulin & Kazaz, 2016). Multiple representations (geoboards or pattern blocks) also helped students' conceptual understanding of geometric shapes (Cass et al., 2003; Kozulin & Kazaz, 2016; Xin & Hord, 2013). For instance, Strickland and Maccini (2012) used linear equations to teach area word problems to secondary students with LD. The researchers also have incorporated cognitive models into their geometry instruction. Xin and Hord (2013) utilized a cognitive model called COMP with a schema to teach perimeter and area problems. This supplemented the concrete-representational-abstract instructional sequence.

The researchers of four studies delivered geometry instructions successfully through video modeling or computer programs, such as virtual manipulatives and LOGO, which is an educational computer program. Cihak and Bowlin (2009), and Satsangi, Hammer, and Bouck (2019) used video modeling to demonstrate problem-solving steps via a laptop computer. Horner (1984) used the LOGO computer program to teach angle-recognition skills. Satsangi and Bouck (2015) used a virtual manipulative program to help students with perimeter and area problems. Only one study included both instructional strategies and educational technology: Satsangi, Hammer, and Hogan (2019) compared the effects of explicit instruction and video modeling when teaching perimeter and area problems to students.

Study Outcomes for Students with LD

The findings of all reviewed studies indicated that interventions improved the general geometry skills of students with LD. Researchers who employed single case designs improved students' performance successfully with medium to very large effect sizes (Tau-U > 0.8) (Cass et al., 2003; Cihak & Bowlin, 2009; Satsangi & Bouck, 2015; Satsangi, Hammer, & Bouck, 2019; Satsangi, Hammer, & Hogan, 2019; Strickland & Maccini, 2012; Xin & Hord, 2013). During the baseline phases or pretests, the participants demonstrated a limited understanding of the geometry concepts that are required to be comprehended by elementary students (e.g., the concepts of perimeter and area). This indicates a low level of geometric thought, based on the van Hiele model (1957). During the intervention phase, the use of instructional strategies and technology provided multiple opportunities for students to observe, understand, and form mental representations of geometric shapes. This instruction supports spatial reasoning and problem-solving skills (Common Core Standards Writing Team, 2013). After the intervention, students with LD improved their geometry outcomes. For example, for perimeter problem-solving skills, three studies provided evidence of mastering the skill of solving perimeter problems with 100% accuracy at the end of the intervention (Cass et al., 2003; Cihak & Bowlin, 2009; Satsangi & Bouck, 2015). Among these three studies, all participants with LD yielded larger effect sizes, ranging from 0.75 to 1.61 (Parker et al., 2011).

The findings also indicated that students with LD who received instruction in groups had positive outcomes with a medium to large effect size (Hedges' g > 0.5 or 0.8). Researchers have utilized quasi-experimental group designs to examine the interventions' effects (Horner, 1984; Kozulin & Kazaz, 2016). In Kozulin and Kazaz (2016), the treatment group of students with LD improved their perimeter and area problem-solving skills significantly when compared to the control group of students without LD ($t = 2.26$, $p < 0.01$). After adjusting for the pretest means for both groups, the effect size for the perimeter problem-solving skills was medium to large ($g = 0.64$; Cohen, 1988) and the effect size for the area problem-solving skills was very large ($g = 1.08$). The results of Horner's (1984) study also reported a medium to large effect size ($g = 0.6$) in angle recognition for students with LD who received geometry intervention. The research results also indicate that students receiving one-on-one or small-group instruction improved their geometry performance, mostly with a large effect size (Tau-U > 0.8). Students with LD receiving instruction in groups showed positive outcomes, with a medium to large effect size.

Study Quality

Liu et al. (in press) evaluated the methodological rigor of geometry interventions based on CEC (2014) standards, and found that the study quality of the geometry interventions for students with LD was generally high. Specifically, the quality of single case designs was relatively higher than that of group designs. Many special education researchers are aware of the necessity of reporting core quality indicators in their studies because studies with poor reporting can affect practitioners, policymakers, grant funders, and journal editors (Talbott et al., 2018).

However, several issues remained in the geometry interventions from this study. For example, several researchers did not describe the training materials used for the intervention or the qualifications of the intervention agents (Satsangi & Bouck, 2015; Satsangi, Hammer, & Hogan, 2019; Strickland & Maccini, 2012; Xin & Hord, 2013). Moreover, one study did not report the fidelity of the implementation throughout the study (Xin & Hord, 2013). Methodologically sound studies provide evidence to establish a functional relation that can be considered to be evidence-based practice (Kennedy, 2005; What Works Clearinghouse, 2017). Researchers should pay attention to quality indicators, conduct high-quality studies that can help close research-to-practice gaps, and make the process of implementation and replication easier (Cook et al., 2015).

INSTRUCTIONAL COMPONENTNS

As indicated in the National Mathematics Advisory Panel report (NMAP, 2008), researchers still need to investigate effective skills and practices that can promote student learning outcomes. In the interest of identifying the separable elements of treatment techniques, researchers have conducted studies to find the instructional components (ICs) that could boost educational outcomes (Swanson & Carson, 1996). Some ICs can be particular learning skills (e.g., rehearsal, imaging, or outlining), self-management activities (e.g., planning or comprehension monitoring), or complex plans that combine several techniques. These instructional variables or instructional strategies have been found to induce educational change (Swanson & Carson, 1996).

Swanson and Carson (1996) used a list of ICs to identify effective teaching approaches (e.g., direct instruction) that were positively associated with improved performance in reading and mathematics for students with LD. Using a meta-analysis of single case designs in a subsequent study, Swanson and Sachse-Lee (2000) found that specific ICs (e.g., small group instruction) caused better academic outcomes for students with LD. Based on their list of ICs, some researchers (Dennis et al., 2016; Kroesbergen & Luit, 2003; Swanson & Hoskyn, 2001; Zheng et al., 2012) successfully identified the effective ICs that were embedded in the instructional models for diverse student groups (e.g., students with mathematics learning disabilities).

Liu et al. (in press) reviewed previous studies, and they constructed a list of 20 ICs for geometry interventions targeting students with LD (Gersten et al., 2009; Swanson & Hoskyn, 1998; 2001). The list included: advance organizers, attributions, control of difficulty or processing demands of tasks, elaboration, explicit practice, large-group learning, novelty in implementing or presenting new teaching materials, one-on-one instruction, peer modeling, questioning, reinforcement, sequencing, skill modeling, small-group instruction, strategy cues, supplements to teacher involvement, task reduction, technology, visual representations, and heuristic strategies (Gersten et al., 2009; Swanson & Hoskyn, 1998; Swanson & Sachse-Lee, 2000). Many of these ICs are aligned with the principles of explicit and systematic instruction, which is an evidence-based practice especially for elementary students with mathematic difficulties (Doabler et al., 2019). The definitions of the ICs are explained in Table 2.1.

Table 2.1

Definitions of Instructional Components

Instructional component	Definition
1. Advance organizer	Statements directing students to look over materials prior to instruction and to focus on particular information, providing information prior to the task, and/or stating the objectives of instruction prior to the task.
2. Sequencing	Statements about breaking down the task, fading prompts, matching the difficulty level of the task to the student, sequencing short activities, and/or using step-by-step prompts.
3. Explicit practice	Statements related to the mastery criteria, distributed review and practice, repeated practice, sequenced reviews, daily feedback, and / or weekly reviews.
4. Questioning strategies (directed response)	Statements related to dialectic or Socratic teaching, the teacher directing the students to ask questions, the teacher and student or students engaging in dialogue, and/or the teacher asking questions.
5. Small-group instruction	Statements about instruction in a small group, and/or verbal interaction between students and/or the teacher, occurring in a small group.

Table 2.1 (continued)

6. Peer-mediated instruction (peer modeling)	Statements about modeling from peers, parents providing instruction, and/or peers presenting or modeling instruction.
7. Modeling	Statements about the processing components or multiple steps related to modeling; simplified demonstrations modeled by the teacher to solve a problem or complete a task successfully; reminders from the teacher to use certain strategies, steps, and/or procedures; think-aloud models; and/or the benefits of taught strategies.
8. Large-group learning	Statements about instruction in large groups, and/or teacher-only demonstrations.
9. Novelty	Statements about the use of diagram or picture presentations, specialized films or videos, instruction via computers, specification that a new curriculum was implemented, and/or emphasis on the teacher presenting new material from the previous lesson.
10. Elaboration	Statements about additional information or explanations provided about concepts, procedures, or steps; and/or redundant text or repetition within text.
11. Reinforcement	Statements about the intermittent or consistent use of probes; daily feedback, fading of prompts and cues; and/or the overt administration of rewards and reinforcers.

Table 2.1 (continued)

12. Control of difficulty	Statements about short activities, a controlled level of difficulty, the teacher providing necessary assistance and simplified demonstrations, the tasks being sequenced from easy to difficult, and/or task analysis.
13. Strategy cues	Statements about reminders to use strategies for multiple steps, the teacher verbalizing the steps or procedures for solving problems, the use of think-aloud models, and/or the teacher presenting the benefits of using strategies or procedures.
14. One-on-one instruction	Statements about activities related to independent practice, tutoring, instruction that is individually paced, and/or instruction that is individually tailored.
15. Supplements to teacher involvement	Statements about homework, and/or parents helping to reinforce instruction.
16. Task reduction	Statements about breaking down the targeted skills into smaller units, mastery criteria, and/or task analysis.
17. Multiple representations	Statements about the students' use of visual representation while solving the problem or the teacher's use of visual representation during the initial teaching, and/or demonstrating the problem-solving process.
18. Technology	Statements about developing pictorial representations, using specific materials or computers, and/or using media to facilitate presentation and feedback.
19. Heuristic instruction	Statements about using a method or strategy that exemplifies a generic approach for solving a problem.
20. Attributions	Statements about the benefits of taught strategies.

In Liu et al. (in press), the authors identified seven ICs used in all nine studies of geometry interventions for students with LD: control difficulty, explicit practice, novelty, skill modeling, strategy cues, heuristic instruction, and multiple representations. (This examination partially responds to Gersten et al.'s [2009] call to analyze ICs in other mathematics topics.) Across the other synthesized studies, other ICs—such as elaboration, one-on-one instruction, sequencing, and task reduction—are commonplace. These components are consistent with the geometry learning model proposed by van Hiele in the 1950s, which emphasizes the importance of sufficient geometry experience (Crowley, 1987). However, interpreting these findings must be done cautiously because students with LD tend to exhibit weaknesses in various mathematics skills, based on their learning needs (Bryant et al., 2000). Therefore, future studies should examine the effectiveness of various ICs, including the ones that have been used less frequently (e.g., small-group intervention).

GEOMETRY VOCABULARY INSTRUCTION

Research has demonstrated that learning mathematical vocabulary has a unique impact on the mathematical performances of fourth-graders (Peng & Xin, 2019). Mathematics verbalizations are positively associated with achievement (Gersten et al., 2009). Students who underperform in mathematics often struggle to decipher mathematical symbols and to communicate accurately using mathematical language (Adams, 2003). The study of mathematics requires both general and discipline-specific vocabulary. Some mathematical terms appear with higher frequency in specific content areas, and have abstract, technical, and densely packed meanings (Bryant et al., 1999; Townsend & Kiernan, 2015).

The CCSSM (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010) require students to deepen their constructed mathematical arguments and reasoning by communicating explanations to others with precise mathematical language. As a core mathematics area, geometry plays an important role in mathematics textbooks and standardized tests. In elementary school, students are expected to identify lines and angles, 2D and 3D shapes, and shapes' attributes (e.g., vertex and side). Therefore, teachers need to provide geometry vocabulary instruction to students using precise language (Fox, 2016).

Even though little research has been conducted on teaching geometry vocabulary to students with LD, some researchers have examined the effects of mathematics language intervention on other mathematics skills using randomized control trials (RCTs) for students with and without disabilities (Hassinger-Das e al., 2015; Powell & Driver, 2015; 2017; Purpura & Reid, 2016). Powell and Driver (2015) examined the effects on first-graders struggling with math of an addition tutoring program with an embedded vocabulary component. The results indicated that students who received both addition and vocabulary tutoring outperformed the control students who did not receive any vocabulary tutoring ($p = 0.048$; Hedges' $g = 0.49$). Furthermore, the students who received only the addition tutoring demonstrated a slight improvement in their vocabulary when compared to students who received addition and vocabulary tutoring. According to Powell and Driver (2015), the possible explanation of this result could be that the mathematics vocabulary was learned through continuous, explicit exposure from individual tutoring or that the role mathematics terms play in addition problems was different compared to the other domain; for example, geometry problems, which involve many difficult or complicated vocabularies. In addition, the authors suggest that mathematics vocabulary instruction probably requires a different instructional framework that needs to be examined.

Storytelling can also help students build their mathematics vocabulary. Two groups of researchers conducted storytelling interventions with pre-school or kindergarten students (Hassinger-Das et al., 2015; Purpura e al., 2016). Purpura et al. (2016) conducted a storybook reading intervention with 47 preschoolers. Their interventions used dialogic reading features in order to examine the impact of a mathematical language program on mathematical knowledge. The interventionists focused on comparative mathematical language (e.g., combine and take away) and spatial language (e.g., near and far). After the intervention, the researchers found that students in the intervention groups significantly outperformed those in the control group on mathematical language ($p = 0.047$, Hedges' $g = 0.42$), as well as mathematical knowledge ($p = 0.049$, Hedges' $g = 0.32$). Similarly, Hassinger-Das et al. (2015) examined a storybook-reading intervention program for 124 kindergarteners with early numeracy difficulties. The program targeted improving both their mathematics vocabulary and mathematics outcomes. Each lesson focused on reading, comprehending, and defining words in the storybook. Their results reveal an immediate intervention effect on mathematical vocabulary in the treatment group in comparison to the other groups, with a significant difference between the treatment group and control group (F (2, 119) = 2.890, $p = 0.06$). The students in the treatment group significantly outperformed the comparison groups (i.e., the number-sense group and the control group) on number sense (Hedges' $g = 0.57$, $p < 0.05$). There is no statistically significant difference in general mathematics achievement between the treatment group and the other comparison groups (F (2, 119) = 3.004, $p = 0.053$).

Teachers using precise mathematical language can be beneficial for students' vocabulary learning (Riccomini et al., 2015). This includes the teacher using formal mathematical language and providing definitions that are developmentally appropriate for the students to understand; for example, using precise mathematical terms and definitions, teaching mathematical vocabulary explicitly, embedding vocabulary instruction in the lesson, and encouraging students to use the vocabulary in context. Hughes et al. (2016) provided suggestions for teaching geometry vocabulary, and they listed examples and non-examples (i.e., examples that are not related to the concepts or attributes being learned) that teachers could use.

SUMMARY OF THE CHAPTER

The purpose of this chapter was to review the studies and methods relevant to geometry interventions for students with LD. Despite the positive findings of the geometry interventions that were conducted with students with LD, several research gaps remain. First, more research is needed to examine the effective geometry instructional strategies for elementary students with LD. Most geometry interventions studied were conducted with secondary students with LD. Second, almost all studies used only one type of researcher-developed measure with no validity or reliability information. Using norm-referenced or standardized tests can help researchers and teachers to evaluate the effects of the intervention and establish the generalizability of the skills students learn. Third, emphasizing geometry vocabulary instruction may be helpful, given that elementary-level geometry introduces many words (e.g., vertices, angles, and polygons). Finally, there is a need for high-quality research that meets the quality indicators of national standards in special education research (Cook et al., 2015).

With limited conceptual and procedural knowledge, students with LD often struggle in mathematics, beginning in elementary school (Miller & Mercer, 1997). However, there are strategies for effective geometry instruction, including using ICs (e.g., guided practice or the use of manipulatives) and emphasizing mathematics vocabulary. The limited evidence available acknowledges that students with LD benefit from receiving geometry interventions.

Therefore, the purpose of this study is to build a knowledge base in the area of geometry for students with LD in mathematics by conducting a single case study that examines the effects of geometry intervention on the geometry knowledge and skills of elementary students with LD. The reason for using a single case design is that, before testing on a large sample size using an RCT, the investigator aimed to collect sufficient evidence on the effects of the geometry intervention using a relatively small sample size for the benefit of the students. In addition, the investigator also used more than one measure in the study to assess and report on student outcomes with reliable validity. In addition, the geometry intervention in the current paper incorporated the teaching of geometry concepts and skills as well as geometry vocabulary to facilitate the learning of geometry skills.

Chapter 3: Methodology

Geometry is a crucial domain within mathematics standards and education (CCSSM, 2010). National and state standards (CCSSM, 2010; TEKS, 2012) have set clear goals to inform geometry instruction for P-12 students. A solid grasp of fundamental geometry concepts and skills is beneficial for students when learning more advanced topics in disciplines such as mathematics, physics, and engineering (Carnevale et al., 2011). The NMAP (2008) recommended that teachers should employ research-based techniques and curriculums to improve elementary students' mathematics performance. However, many students with learning disabilities (LD) experience various problems when learning mathematics, including a lack of basic geometry concepts and skills (e.g., Cass et al., 2003; Kozulin & Kazaz, 2016). Even though researchers have found that geometry interventions significantly improved the geometry outcomes for participants with LD (e.g., Hord & Xin, 2015; Satsangi et al., 2015), most of the studies were conducted with junior and high school students. Very few studies included elementary students with LD. Therefore, the purpose of this study is to determine the effectiveness of a geometry intervention on the performance of students with LD in the fourth and fifth grades. Guiding this study were four research questions:

1. What is the immediate effect of a geometry intervention on the geometry performances of fourth- and fifth-grade students with LD as measured by a proximal measure (adapted easyCBM)?
2. To what extent do the fourth- and fifth-grade students with LD maintain their geometry performance one week after the conclusion of the intervention, as measured by a proximal measure (adapted easyCBM)?

3. To what extent do the fourth- and fifth-grade students with LD generalize their geometry knowledge to a distal measure (KeyMath-3 geometry subtest)?
4. What are the perspectives of the fourth- and fifth-grade students with LD on the geometry intervention?

To conduct research in an elementary school, before beginning the study, the investigator obtained approval from the Institutional Review Board at the University of Texas at Austin, approval from the elementary school's management board, and signed parental permission forms and student assent forms. The investigator explained the study to teachers, parents, and participants when they raised questions.

PARTICIPANTS

The participants were elementary students who were school-identified with LD from the fourth and fifth grades. The sample size was originally four participants- three 4th graders and one 5th grader. The investigator worked with the principal and the mathematics teachers to identify potential participants. Then, the students were given the opportunity to take consent forms home to their parent or legal guardian. The parental permission forms and the student assent forms were in English.

The emergence of a new, severe respiratory coronavirus called COVID-19 has been causing a global pandemic across the world since December 2019, including the U.S. (Lai et al., 2020). Before the widespread school shutdown in late March 2020, the study took place at the University of Texas at the Austin Elementary Charter School (UTES), where the participants received daily instruction. At the time of the study, 66% of the UTES student population was Hispanic, 16% was Caucasian, 12% was African American, 2% was Asian, and 4.6% was multiethnic. Almost one-quarter of the students (22%) were nonnative English speakers, and more than half of them (55%) had low socio-economic backgrounds.

The UTES closed after the spring break in 2020 to better protect the health and well-being of the students, staff, families, and communities. Therefore, the geometry intervention was delivered in-person for one month and then moved online after the spring break. All participants had to receive the instruction for the later part of the geometry intervention remotely through Zoom, which is an online videoconferencing platform. Only three participants were able to continue and completed the project after the investigator communicated with the parents with the help of the UTES principal.

INCLUSION CRITERIA

To identify the participants, the investigator used a two-gate screening procedure in the participant-selection process (Brendefur et al., 2018). In Gate 1, the investigator worked with the elementary school principal to select potential participants with LD (in the fourth and fifth grades). Based on the results of the Pearson Education End of Year test in 2019, the investigator located several students who scored below the proficiency level on mathematics (with a less than 70% accuracy rate). The Pearson Education End of Year test is a school-administered test implemented at both the beginning and the end of the school year. The items tested are aligned with Texas education standards. The principal worked with the investigator by getting access to the elementary school system and providing a list of potential participants to the investigator based on the Gate 1 criterion.

In Gate 2, the investigator administered a geometry test using third-grade easyCBM geometry (original version) with the potential participants determined using Gate 1. The reasons for using third-grade easyCBM geometry were as follows: (a) no easyCBM geometry measure was available for the fourth grade; (b) the fourth-grade CCSSM version of easyCBM only included limited items related to geometry, which made it hard to examine students' geometry ability; (c) the third-grade easyCBM geometry incorporates items with the fundamental concepts and skills needed for fourth-grade geometry. Therefore, the investigator chose the third-grade easyCBM geometry measure. Participants who performed below or at the 25th percentile on the measure were eligible. In total, six participants were below the 25th percentile, and the investigator chose the lowest four participants. The fourth and fifth lowest students did not tie on their scores. Students who were English-language learners at school were excluded by Gate 1.

The information on participant demographics is in Table 3.1. The information includes the age, grade level, gender, ethnicity, disability type (including the identification method), reduced-price lunch status, home language, universal-screener results, and pretest results. The investigator stored all the data on a secure Category 1 server at the College of Education of the University of Texas at Austin.

Table 3.1

Participant Demographic Information

Variable	**Andy**	**Peggy**	**Charlie**
Age (Years. Months)	10. 8	9. 7	10. 4
Grade	5	4	4
Gender	Female	Male	Male
Ethnicity	African American	African American	African American
Disability	School-identified LD, math calculation & dyslexia	School-identified LD, math & written expression	School-identified LD & math calculation

Table 3.1 (continued)

Free or reduced lunch	Y	Y	Y
Home language	English	English	English
Universal screener	BP At or below 18%*	BP At or below 21%*	BP At or below 24%*
Pretest	11** lowest 10%	11** lowest 10%	11** lowest 10%

Note. Age was reported upon the reception of the consent and assent forms in Jan 2020; Y = yes; N = no; BP = below proficiency level; * = based on the Pearson Education End of Year test in 2019; ** = third-grade easyCBM geometry test with 16 test items.

INTERVENTIONIST AND SETTING

The investigator was the interventionist. The investigator has five years of K-8 lead-teaching experience in a private school. The investigator has worked in inclusive classrooms, teaching students including those with LD. She earned a master's degree in Early Childhood Special Education, and she is pursuing a doctoral degree in Special Education focusing on Learning Disabilities and Behavioral Disorders (LD/BD).

The setting of the intervention consisted of two parts: in-person instruction and online instruction. During the in-person instruction period (before the COVID-19 pandemic), the study took place in a quiet conference room at the elementary school the participants attended, and it occurred before or after school hours (e.g., 7:40 am to 8:00 am, or 3:30 pm to 4:50 pm). The room was equipped with a table, chairs, a whiteboard, and dry-erase markers and erasers. However, the intervention moved online after the beginning of the COVID-19 pandemic in April, 2020. The participants received instruction at home through Zoom. The intervention time was based on each students' study schedule after discussing with the teacher and parents (e.g., 10 am). In total, the intervention sessions lasted 10 weeks for the three participants. For each participant, the intervention took about three weeks, with four or five 30-minute sessions per week for a total of seven lessons (some lessons were split over more than one session, if needed).

RESEARCH DESIGN

The investigator used a single case multiple probe design to examine the effects of a geometry intervention on the geometry concepts and skills of fourth- and fifth-graders with LD. Single case designs (SCD) involve the repeated measurement of behavior over time or across settings (Kennedy, 2005), which allows for the detailed analysis of individual geometry outcomes. SCD allow researchers to identify evidence used to develop special education best practices (Rodgers et al., 2017; Tawney & Gast, 1984). Researchers have used SCD for more than 50 years across a variety of fields, such as psychology and special education (Horner et al., 2005; Ledford et al., 2018).

The multiple probe design is a type of multiple baseline design. Multiple baseline designs demonstrate experimental control by establishing at least three concurrent baselines (What Works Clearinghouse, 2017). However, a multiple probe design is considered to be a more efficient method than a multiple baseline design because researchers can collect and analyze data intermittently and systematically in each session and phase (Gast, 2010; Horner & Baer, 1978). The investigator introduced the intervention sequentially to the participants so that patterns of behavioral change across different participants could be observed (Kennedy, 2005). The data collected for this single case design was defined, measurable, and recorded physically.

There were three replications of the experimental control to establish the functional relations between the geometry intervention and the learning outcomes of the participants in this study. During the baseline phase, the investigator collected data intermittently but consistently across the three participants. The baseline data was used to establish the initial geometry performance of each participant. When the first participant (participant 1) reached a stable baseline (e.g., no increasing trend in the adapted easyCBM geometry scores), the investigator introduced the geometry intervention to participant 1 while the rest of the participants remained in the baseline phase. The data points collected from participant 1 right before and right after the intervention allowed the investigator to examine the level of change in the dependent variable (DV) at each time point. Therefore, the functional relation between the DV and the independent variable (IV) was observed across the participants throughout the study by consistently introducing and manipulating the IV at different time points (Kennedy, 2005). After the intervention had been completed for each participant, the maintenance and generalization phases followed. The maintenance phase was one week after the intervention. The generalization test was after the maintenance phase.

Independent Variable

The IV is the geometry intervention. The investigator developed a 7-lesson geometry intervention. The intervention was designed for in-person instruction; however, it was switched remotely in the middle of the intervention due to COVID-19. To complete the seven lessons, each participant needs to spend approximately 2.5 weeks. Each lesson lasted 1-2 sessions depending on the students' performance on the practice problems. For each session, the investigator implemented the intervention for 30 minutes.

During the intervention development process, the investigator accessed commercially-available mathematics curricula and other geometry materials and used them as guiding tools. Advice from the academic advisor was also actively sought, having received feedback on the critical lesson structures (e.g., the use of Warming up before Interactive modeling) and the sequence of the intervention topics. To identify the intervention topics, the investigator listed the standards-aligned topics based on the CCSSM and Texas Essential Knowledge and Skills (TEKS) for fourth-grade geometry in Table 3.2.

After three iterations, the geometry intervention was finalized with seven lessons before the intervention started. The topics of the lessons was in a specific sequence: the geometry concept taught in earlier lessons laid the foundation for the concepts taught later in the lessons. Specifically, the intervention topics included parallel lines and perpendicular lines, angles, properties of 2D shapes (e.g., vertices and sides), perimeter and area problems for regular shapes (e.g., squares, rhombuses, pentagons, and hexagons) and irregular shapes (those with only right angles, see Appendix A), and symmetry lines.

Table 3.2

Lesson Sequence and CCSSM & TEKS Alignment

Lesson	Lesson content	CCSSM	TEKS
1	Parallel lines and perpendicular lines	CCSS.MATH. CONTENT.K.G.A.1	111.6. Mathematics 4.6(A)
2	Angles	CCSS.MATH. CONTENT.4.G.A.1	111.6. Mathematics 4.6(C) & 4.7 (D,E)
3	2D shapes	CCSS.MATH. CONTENT.4.G.A.2	111.6. Mathematics 4.6(D)
4	Perimeter	CCSS.MATH. CONTENT.4.MD.A.3	111.6. Mathematics 4.5(D)
5	Area	CCSS.MATH. CONTENT.4.MD.A.3	111.6. Mathematics 4.5(D)
6	Symmetry lines	CCSS.MATH. CONTENT.4.G.A.3	111.6. Mathematics 4.6(B)
7	Review	NA	NA

Note. NA = not applicable.

The geometry intervention included evidence-based ICs and geometry vocabulary instructions. Each lesson included five sections based on the evidence of effectiveness in previous research (Bryant et al., 2020): warming up, interactive modeling, guided practice, review, and independent practice. The sample lesson is in Appendix B.

Warming Up

The first section of each lesson was warming up. The activities in the warming up section helped the investigator to understand the participants' prerequisite skills with shapes and their geometric-thinking level (van Hiele-Gedolf, 1957). The investigator spent about five minutes in this section. For example, one activity is called "finding the target shape." The investigator provided a practice sheet with many different shapes and asked the student to find the target shape. A sample question was "Can you find how many triangles there are in this picture?" The student answered the question by either tracing the shapes using a colored marker or responding verbally. After the student finished, the investigator provided the correct answers and got the student to check his or her own responses.

Interactive Modeling

Interactive modeling was the second section, which lasted about 15 minutes. The investigator scaffolded the geometry concepts by introducing the ideas and modeling of the key procedures to solve the problems. Before introducing the definition and attributes of the geometry concepts, the investigator used real-life examples and talked about the importance of learning the shape based on the setting. For example, in lesson 1, the investigator started the interactive-modeling section by saying, "Today, we are going to learn about parallel lines. When you look at the window, the top and bottom lines are parallel. Can you find some other examples of parallel lines in this room? Point them out to me." The investigator also modeled the ways to identify 2D shapes by talking about the attributes (e.g., sides, angles, and vertices of the target shape). For example, in lesson 3, the participant was asked to find the number of sides of the triangles. The investigator modeled this by saying, "This question is asking you to find the number of angles. Here is how I do it; I will start counting the angles from here. One, two, and three! There are three sides in this triangle. A triangle can be classified based on the types of angles it has; for example, this triangle is called a right triangle because it has one right angle in it."

The investigator also used questioning strategies to provide opportunities for the participants to respond. The student was exposed to many different examples of geometry models, either in concrete (e.g., a pattern block) or in pictorial (e.g., pictures of triangles) forms. For example, the investigator pointed at a right triangle and asked questions, such as "What is the name of this figure?"; "Why do you think it is a [triangle]?"; "What do you notice about this shape?"; "Does this triangle have three sides?"; "Does this triangle have right angles?"; or "How many right angles are there in this shape?" Through answering the questions and observing the target shapes multiple times, the student could deepen his or her understanding of the shape and get ready to solve geometry problems in the next section.

Guided Practice

This section followed the interactive-modeling section and provided student-practice opportunities. The investigator used flash cards to get the student to practice the geometry words by recalling the shape names from the shape pictures; this lasted for two minutes. After the activity, the student learned to solve the perimeter problem for the shape. For example, in lesson 1, the investigator provided two triangle problems, with triangles in different orientations and with different sides and angles. For the first problem, the student practiced solving the perimeter problems by knowing the lengths of all sides. The investigator checked the student's work and asked the student to explain the procedures used to solve the problems. After the student responded, the investigator provided feedback on the student's answers. If the student had no issues with solving the first problem, the investigator asked the student to move on to the second problem, in which the student would find the length of the missing side of a triangle using the value of the perimeter and the lengths of the two sides given.

Check-up-error Analysis and Review.

In this section, the investigator and participants reviewed and summarized their learning from each lesson for about five minutes. The investigator clarified any misconceptions the participant displayed. Participants could take notes in their journals by writing down new geometry vocabularies using the adapted Frayer model.

Independent Practice

In this section, the participants practiced solving geometry problems without guidance from the investigator. Each participant had five minutes to complete four questions. The practice sheet was researcher-developed, based on the lesson objectives. The items were related to the concepts introduced in each lesson; for example, shape identification, and knowledge of shapes' sides, angles, and perimeter. By checking the students' performances, the investigator identified student error patterns and provided additional instructions if needed.

Instructional Components

The investigator incorporated different ICs with various teaching tools to reinforce the understanding of geometry concepts and vocabulary learning; for example, pattern blocks, flash cards, AngLegs, and a notebook with practice questions.

The geometry intervention also included several major ICs: sequencing, practice opportunities, questioning strategies, mathematically precise language, modeling, feedback, scaffolding, elaboration, multiple presentations, mathematical reasoning, mathematical connections, and reinforcement. Those ICs were embedded into the daily lessons through teaching and activities. The definition of each instructional component of this study is available in Table 2.1 in Chapter 2.

Vocabulary Instruction

The geometry intervention incorporated vocabulary instruction using the Frayer model and flash cards. The Frayer model is a graphic organizer that provides a thorough understanding of new words (Foster, 2007; Frayer et al., 1969). This model has been used in reading interventions. The model includes five sections, including the vocabulary (the spelling of the word), definition, characteristics, examples, and non-examples of the target geometry term (see Figure 3.1). Using visual cues (i.e., examples and non-examples of the shapes) can be beneficial for teaching mathematics vocabulary (Bruun et al., 2015). Through this model, students can build a broad and in-depth understanding of the geometry terms by becoming aware of the shapes' attributes by drawing shapes. The investigator asked the students to write down the new geometry terms at the end of each lesson in a student notebook, and they reviewed the terms in the following sessions. When the intervention was finished, the student completed the journal with the key geometry vocabulary they had learned.

Existing research also indicated that using flashcards is more effective in teaching vocabulary than a word list, because flashcards were more effective for teachers to use for demonstrations in a class activity with learners (Komachali & Khodareza, 2012; Sitompul, 2013). In this geometry intervention, the participants had multiple practice opportunities, through flashcard activities to memorize geometry words in each session.

MATERIALS

Table 3.3 displays the materials needed for the geometry intervention. During the screening, baseline, intervention, maintenance, and generalization phases, the investigator administered the proximal and distal measures (e.g., adapted easyCBM geometry and, KeyMath-3 geometry subtest) using different teaching materials.

Table 3.3

Materials of the Geometry Intervention

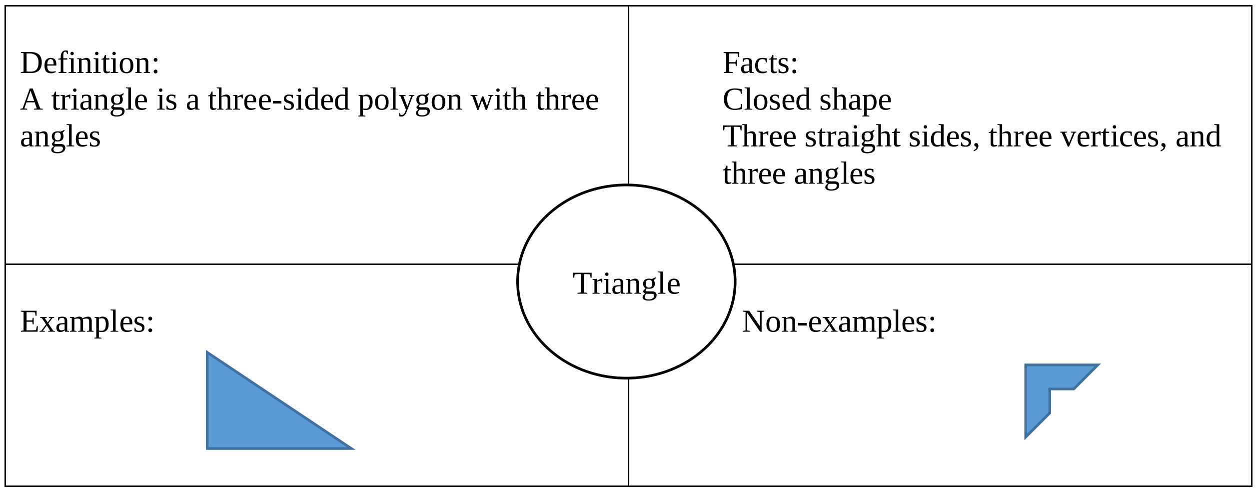
Phases	Instructional materials	Student materials
Screening	1. Third-grade easyCBM geometry measure 2. Third-grade easyCBM test administration sheet 3. Stopwatch	1. Pencils and erasers 2. Calculators

Table 3.3 (continued)

Baseline	1. KeyMath-3 geometry test book 2. KeyMath-3 geometry administration sheet 3. Adapted easyCBM geometry measure 4. Adapted easyCBM test administration sheet 5. Stopwatch	1. Pencils and erasers 2. Calculators
Intervention	1. Geometry lessons 2. Markers and erasers 3. Pattern blocks 4. AngLegs 5. Addition and subtraction fact sheet 6. Flash cards with geometry vocabulary 7. Adapted easyCBM geometry measure 8. Adapted easyCBM test administration sheet 9. Social validity form 10. Fidelity checklist 11. Stopwatch	1. Pencils and erasers 2. Colored markers 3. Calculators 4. Student notebooks
Maintenance	1. Adapted easyCBM geometry measure 2. Adapted easyCBM geometry administration sheet 3. Stopwatch	1. Pencils and erasers
Generalization	1. KeyMath-3 geometry test book 2. KeyMath-3 geometry administration sheet	1. Pencils and erasers

Figure 3.1

Sample Frayer Model



Dependent Variables

There were four DVs (see Table 3.4) used, and the description of each DV is also provided. The DVs are as follows: (a) student geometry outcomes on the proximal measure (adapted easyCBM geometry); (b) student geometry outcomes one week after the end of the intervention on the proximal measure (adapted easyCBM); (c) student geometry outcomes on the distal measure (KeyMath-3 geometry subtest); and (d) student perspectives on the geometry intervention. Each DV is aligned with the corresponding research question(s).

Research Question 1: Participant outcomes

Answering the first research question involved examining the immediate effect of the IV (a geometry intervention). The DV is the performance of the participant on the proximal measure (adapted easyCBM geometry) during the baseline and intervention phases. The results of the adapted easyCBM geometry were collected and analyzed to determine the changes in the geometry learning outcomes. The geometry intervention's effectiveness was evaluated through visually analyzing the graphical data and calculating the effect sizes using the non-overlap of all pairs (NAP) method (Horner et al., 2005; Kennedy, 2005; Kratochwill et al., 2010; Parker & Vannest, 2009). The concepts and skills examined using the adapted easyCBM geometry included the following: parallel lines and perpendicular lines, angles, properties of 2D shapes, perimeter problems, area problems, and symmetry lines.

Research Question 2: Maintenance effect

The second DV corresponds to the second research question, which is regarding the maintenance of the geometry concepts and skills. During the maintenance phase (one week after the intervention phase), the participants completed a proximal measure (adapted easyCBM geometry). The participants' performances on the measure helped to answer the second research question. To evaluate the maintenance effects, the investigator analyzed the difference in the results between the intervention phase and maintenance phase.

Research Question 3: Generalization effect

The third DV involved examining the degree to which the learned skills from the geometry intervention was generalized to a distal measure (KeyMath-3 geometry subtest). The analysis of the participants' performances on KeyMath-3 geometry helped the investigator answer research question 3, regarding the generalization of the geometry intervention.

Research Question 4: Social validity

The last DV relates to the participants' perspectives of this study. All participants were asked to complete a social validity form after the intervention phase. The research-developed social validity form contained rating-scale questions on the strengths and weaknesses of the intervention. The information collected in the forms helped the investigator to answer research question 4.

MEASURES

Three measures were used in the geometry intervention: the adapted easyCBM geometry measure, a KeyMath-3 geometry subtest, and the social validity form. Table 3.4 shows the measures and the administration time of the measures for each research question.

Table 3.4

Research Questions, Dependent Variables, and Measures

Research question	DVs	Measures	Administration time
RQ1 1. What is the immediate effect of a geometry intervention on the geometry performances of fourth- and fifth-grade students with LD as measured by a proximal measure (adapted easyCBM)?	Student geometry outcomes on the proximal measure	Adapted easyCBM geometry	Baseline and intervention phases

Table 3.4 (continued)

RQ2: To what extent do the fourth- and fifth-grade students with LD maintain their geometry performance one week after the conclusion of the intervention as measured by a proximal measure (adapted easyCBM)?	Geometry outcomes on the proximal measure	Adapted easyCBM geometry	Maintenance phase
RQ3: To what extent do the fourth- and fifth-grade students with LD generalize their geometry knowledge to a distal measure (KeyMath-3 geometry subtest)?	Geometry outcomes on the distal measure	KeyMath-3 geometry subtest	Pretest: before the baseline phase Post-test: generalization phase
RQ4: What are the perspectives of the fourth- and fifth-grade students with LD on the geometry intervention?	Students' perspectives on the intervention	Researcher-developed social validity form	After the intervention phase

Note. RQ = research question.

Screening and Proximal Measure: EasyCBM Geometry

The geometry intervention included the use of a third-grade easyCBM geometry measure (See Appendix C; Alonzo et al., 2010) during the screening process. The easyCBM mathematics tests were developed to assess students' knowledge and mastery of skills as outlined in the NCTM focal-point standards and the CCSSM. First used in 2009, the easyCBM measure has been given to diverse student populations across the US.

Usually, the test includes 16 items, and it should take 15 minutes for most students to complete.

Table 3.5

EasyCBM Geometry Score Interpretation Table

Percentile	***Geometry***		
	Fall	**Winter**	**Spring**
10th	10	11	12
25th	11	12	13
50th	13	13	14
75th	14	15	15
90th	15	16	16

Based on the instructions in the manual, Table 3.5 presents the scores from G3 easyCBM geometry for the student percentiles based on three time periods in a school year. The technical report for the easyCBM measure for grades K-8 is consistent internally with a Cronbach's alpha of 0.90, and split-half reliabilities of 0.80 (first half) and 0.86 (the second half) across all 18 mathematics measures (Wray et al., 2014).

For progress monitoring purposes, alternate fourth-grade adapted easyCBM geometry measures were used during the baseline phase, intervention phase, and maintenance phase. Because no fourth-grade easyCBM geometry measure was available, according to the easyCBM website (https://easycbm.com/), the investigator developed adapted easyCBM geometry measures by selecting the test items from the fourth-grade

CCSSM easyCBM measures (see Appendix D for sample measure). There were four alternate forms used at equivalent difficulty levels to keep track of the students' progress (e.g., the adapted easyCBM form 1 and the adapted easyCBM geometry form 2).

Distal Measure: KeyMath-3 Geometry

The investigator also administered the KeyMath-3 geometry subtest with 36 items to examine the generalization of the intervention. The KeyMath-3 measure is a norm-referenced diagnostic test on mathematical concepts and skills for individuals ranging in age from 4 years and 6 months old to 21 years and 11 months old (Connolly, 2010). KeyMath-3 covers a broad range of mathematical topics, and it is linked to the NCTM (2000) *Principles and Standards for School Mathematics*. KeyMath-3 covers three content areas: basic concepts, operations, and applications. The geometry subtest is one of the tests under basic concepts in KeyMath-3. The assessment items include knowledge of spatial relationships, spatial reasoning, coordinates, symmetry, and geometric modeling. The test provided the investigator with data on the abilities of the participants to analyze, describe, compare, and classify 2D and 3D shapes.

According to the KeyMath-3 manual (Connolly, 2007), there is evidence to show that the measure has good psychometric properties. The internal-consistency reliability of KeyMath-3 is 0.96. The alternate-form reliability is 0.96. The test-retest reliability is 0.97. The test was developed with the help of educational researchers and practitioners, and it was aligned with national standards. There are also high correlations between KeyMath-3 and other popular instruments in the US. For example, for kindergarten through fifth-grade the correlation between KeyMath-3 and the Kaufman Test of Educational Achievement

(Second Edition) is 0.83, and the correlation between the KeyMath-3 and the group mathematics assessment and diagnostic evaluation is 0.82.

The investigator used the normative and interpretative tables of Keymath-3 to report the students' performance on the geometry subtest. There are two ways to describe student performance: relative standing scores (i.e., scale scores, standard scores, and percentile scores) and developmental scores (i.e., grade equivalents, age equivalents, and growth-scale values). The interpretation of the normative scores is available in the KeyMath-3 manual (Connolly, 2007). The descriptive categories of scale scores, standard scores, percentile ranks, and standard deviations from the mean are presented in Table 3.6.

Table 3.6

Descriptive Categories of KeyMath-3 Outcomes

Descriptive category	***Scale score range***	***Standard score range***	***Percentile rank range***	***Standard deviations from the mean***
Well-below average	4 or below	70 or below	2 or below	−2.0 or below
Below average	5–7	71–85	3–16	−2.0– (−1.0)
Average	8–12	86–114	17–83	−1.0–1.0
Above average	13–15	115–129	84–97	1.0–2.0
Well-above average	16 or above	130 or above	98 or above	2.0 or above

Social Validity

Social validity involves examining the experience of each participant in relation to the intervention. The investigator used a social validity form to collect information about the importance of, effectiveness of, and satisfaction with the geometry intervention after it had been completed (see Appendix E). The measure consisted of nine questions on a five-point Likert scale. Participants scored “1” to indicate they strongly disagree with the statement, “2” if they disagree, “3” if they feel neutral, “4” if they agree, and “5” if they strongly agree. The investigator analyzes and reports on the results of students’ responses in Chapter 4.

PROCEDURE

The design of the geometry intervention was a multiple probe design with three participants. The delivery of the intervention switched from in-person to remote. After screening the participants, the intervention started with the baseline phase and moved on to the intervention, maintenance, and generalization phases. The systematic and sequential introduction of the IV (i.e., geometry lessons) took place once the first participant had reached a stable baseline with three consecutive data points.

The participants received the intervention four to five times a week. After the participants had met the screening criteria, the participants entered the baseline phase. From session 1 to session 4, all participants took baseline probes (adapted easyCBM geometry). On session 5, the first participant (Andy; all the names used in this dissertation are pseudonyms) reached a stable baseline and entered the intervention phase; however, the rest of the participants remained in the baseline phase and took baseline probes on the same session. Starting from session 5, Andy received geometry intervention five times a week for two weeks and took the weekly intervention probes. Andy completed six lessons before the school shutdown.

While Andy received geometry intervention, the other participants in the baseline took the baseline probes weekly. When Andy's geometry outcomes demonstrated a consistent pattern of intervention effects (e.g., levels, trend, and variability), the second participant (Peggy) with a relatively stable baseline entered the intervention phase. It was planned that, when choosing the participants to enter intervention phases, if more than one participant reached stability in the baseline probes at the same time, the investigator would choose one participant randomly to enter the intervention phase next. Therefore, even though participant 3 also reached a stable baseline, the investigator picked Peggy as the second participant to enter the intervention phase. Peggy finished three lessons before the school closure. The rest of the lessons (lesson 4 to lesson 7) were received online after the spring break. When Peggy showed effects from the intervention, the third participant (Charlie) with a stable baseline started the intervention after the school shutdown. Charlie is the only participant who received the intervention completely online.

When each participant finished the intervention, he or she entered the maintenance phase and took a maintenance probe (adapted easyCBM geometry) one week after the conclusion of the last intervention session. After the maintenance phase, each participant entered the generalization phase by taking a post-test of KeyMath-3 geometry. Table 3.7 provides details of the test administration in each phase.

Table 3.7

Timeline of the Testing Activities

Phase	Testing activities
Screening	• Third-grade EasyCBM geometry test (15 min)
Baseline	• KeyMath-3 geometry subtest (30 min) • Fourth-grade adapted easyCBM geometry test (10 min); at least three data points for each participant
Intervention	• Six intervention lessons (14 sessions); one review lesson (two sessions); and four or five sessions per week (30 min per session) • Fourth-grade adapted easyCBM geometry (10 min) • Social validity form (5 min)
Maintenance	• Fourth-grade adapted easyCBM geometry test (10 min)
Generalization	• KeyMath-3 geometry test (30 min)

Baseline Phase

During baseline phase, the investigator employed the multiple probe design to collect data intermittently. The first baseline probe was conducted the day after the screening test. The students completed a 10-minute baseline probe (adapted easyCBM geometry) outside their classrooms. The investigator provided them with answer sheets, pencils, scratch paper, and calculators if needed (if the student's IEP goal indicated the need for a calculator). First, the student listened to the instructions for the test. Then the investigator set a timer for 15 minutes and started the test. The student had 15 minutes to complete the 16 items. After the student had completed the test, the investigator collected the answer sheet and scored the test by determining the accuracy of the answers. A minimum of three data points for baseline probes were collected for each participant before moving them to the intervention phase. When the performance of a participant reflected a stable level, the intervention phase began.

Intervention Phase

During the instructional time, the investigator used the instructional materials described in Table 3.3 to implement the intervention plan. A sample lesson is presented in Appendix E.

In the intervention phase, the investigator provided each participant with one-on-one geometry instructions, either in person in a chosen room or online using Zoom. The intervention was delivered based on the student's schedule (see Appendix G notes section). The instructional procedure applied to all participants. During the intervention phase, the introduction of the IV (i.e., the geometry intervention) was staggered across the three participants. Each participant received the intervention after demonstrating a stable baseline in terms of their level and trend. For example, when one participant demonstrated stability on the baseline probes, the investigator introduced the intervention, but the other participants needed to remain in the baseline phase. When the second participant reached a stable baseline, the intervention began for the second participant. The examination of the data points before and after the intervention allowed the investigator to detect the intervention effects.

Post-intervention Phase

After the intervention phase, the participants completed a generalization test using a distal measure (KeyMath-3 geometry subtest). The distal measure took about 30 minutes to complete, following the end of the intervention. The investigator first provided the instructions for the generalization test and used the testing materials to implement the test. The participant answered the questions in the test book. After a participant answered a question, the investigator wrote down whether the answer was correct or incorrect on the KeyMath-3 geometry answer sheet, and then moved on to the next question.

Maintenance Phase

Maintenance probes were used to determine whether the participants were able to retain the learned concepts and skills for a longer period. To determine the maintenance of the intervention, each participant took an adapted easyCBM geometry test. The investigator administered the maintenance probe one week after the intervention was complete. The test administration procedures and conditions for the maintenance tests were identical to those of baseline and intervention tests. After all tests were completed, a social validity form was provided to the participants.

TREATMENT INTEGRITY AND INTER-SCORER AGREEMENT

Treatment Integrity

The investigator created a multi-dimensional fidelity checklist for the geometry intervention. A fidelity checklist helped to collect data about whether lessons were delivered as planned. It is an observational tool used to record the occurrence of evidence-based instructional practices, and it is based on a systematic review of the research (Gagnon & Maccini, 2005; Gersten et al., 2009). The adapted fidelity checklist included different intervention behaviors during mathematics instruction (e.g., checking for understanding, explicit feedback, questioning strategy, and promoting student dialogue; see Appendix F). The fidelity check happened regularly throughout the implementation for 28.57% of the intervention (the beginning, middle, and end of the intervention).

The observers completing the fidelity checklist were trained on interrater reliability (IRR). To calculate the IRR, the investigator divided the number of agreements by the total number of agreements plus disagreements and then multiplied by 100. The investigator also used Cohen's kappa (Cohen, 1960) to correct for agreement rates that might occur by chance. The investigator calculated the kappa value based on previous research (Cohen, 1960; McHugh, 2012). The kappa value is interpreted as poor if it is below 0.40, fair if the value is between 0.40 and 0.59, good if the value is between 0.60 and 0.75, and excellent if the value exceeds 0.76 (Cicchetti, 1994). Before the research begins, a high IRR (IRR > 80%) between two observers needs to be reached. A high kappa value (kappa > 60%) is expected for this study. Disagreement may occur and needs to be resolved for consistent data collection. To maintain the high methodological rigor of this study, observations of the implementation involved fidelity checks throughout the course of the intervention for all participants (Cook et al., 2015).

Inter-scorer Agreement

The investigator scored all the measures across the screening, baseline, intervention, maintenance, and generalization phases. Another scorer scored the measures again independently. The inter-scorer agreement was calculated using the number of agreements in participants' responses divided by the total number of agreements and disagreements, then multiplied by 100 (Johnson & Semmelroth, 2012). The inter-scorer agreement was expected to be 90% or higher for the geometry intervention so that the result would be considered to be highly acceptable (Neuendorf, 2002).

DATA ANALYSIS PLAN

Visual Analysis

One traditional approach for the data analysis of single case designs involves visual analysis (Lane & Gast, 2013). Visual analysis is a method used to evaluate the evidence of a functional relationship between an IV and an outcome variable through interpreting (a) the level, (b) the trend, (c) the variability, (d) the immediacy of the effect, (e) the overlap, and (f) the consistency of data patterns across similar phases, regardless of the type of single case design (Horner et al., 2005; Kennedy, 2005). Through visual analysis, a study can be categorized as positive, negative, or mixed (Lang et al., 2012). If the data from the graphs displays an improvement trend for all of the DVs related to the geometry outcomes for all participants with LD in the study, then the study will be marked as positive. If the study shows no improvement for any participant with LD on any DV related to geometry, it will be marked as negative. If only some participants with LD improved, the study will be marked as mixed.

The investigator displayed the participants' results visually (Kennedy, 2005) by graphing data points for each individual participant. The data patterns helped to determine the study's next step. For example, the investigator decided on the time to introduce the intervention based on the student's baseline performance. In this study, three between-phase patterns and multiple within-phase patterns were demonstrated (Kennedy, 2005).

Effect Sizes

Researchers disagree on the best quantitative-analysis methods for single case designs (Hedges et al., 2013; Parker et al., 2011). In this study, the investigator employed the NAP to examine the effects of the geometry intervention and report on the participants' geometry performances. The NAP compares the extent to which the data points overlap between two phases. The NAP is considered to be an improvement on the methods of calculating the effect size for single case designs (Parker & Vannest, 2009). According to Parker and Vannest (2009), a value at or below 0.65 is considered to be a weak effect size, between 0.66 and 0.92 is a medium effect size, and between 0.93 and 1 is a strong effect size.

Research Question 1

The first DV has been used to examine the progress participants made in learning geometry concepts and skills by receiving the geometry intervention. By graphing and analyzing the participant data, the investigator examined the participant probes of the proximal and distal measures in each phase (e.g., the baseline and intervention phases). The investigator also compared the levels of the last three data points in the baseline phase to the levels of the first three data points in the intervention phase to examine the immediate effect of the intervention (Kennedy, 2005). Visual analysis and effect size were both used to answer research question 1.

Research Question 2

To answer research question 2, the investigator administered the adapted easyCBM geometry test to determine the maintenance effect of the geometry intervention on the participants. Specifically, the maintenance effects of the geometry interventions were evaluated using a visual analysis of the graphical data procedures to compare the baseline, intervention, and maintenance phases (Horner et al., 2005; Kennedy, 2005).

Research Question 3

The investigator analyzed and compared the pretest and post-test results of the distal measure, which are the results from the KeyMath-3 geometry subtest before and after the intervention. A comparison of the raw scores and scale scores for the KeyMath-3 geometry before and after the intervention indicate how the participants could generalize the learned skills to other problems.

Research Question 4

The investigator collected the students' answers from the students' social validity forms and calculated the mean score of each item across the three participants. The mean score reveals the participant's overall perspective. For example, a mean score of 4.5 for an item shows that the participants strongly agree with the statement of the item.

Chapter 4: Results

The geometry intervention in this paper focuses on teaching standards-aligned geometry concepts and skills to students with LD at the elementary level, specifically for students in the fourth and fifth grades. The purpose of this study is to investigate the effects of a geometry intervention on the geometry performances of three participants. The research questions are as follows:

1. What is the immediate effect of a geometry intervention on the geometry performances of fourth- and fifth-grade students with LD as measured by a proximal measure (adapted easyCBM)?
2. To what extent do the fourth- and fifth-grade students with LD maintain their geometry performance one week after the conclusion of the intervention as measured by a proximal measure (adapted easyCBM)?
3. To what extent do the fourth- and fifth-grade students with LD generalize their geometry knowledge to a distal measure (KeyMath-3 geometry subtest)?
4. What are the perspectives of the fourth- and fifth-grade students with LD on the geometry intervention?

This chapter includes the results for each question, followed by a chapter summary. The investigator also reports on the fidelity of the intervention. It should be noted that four participants' parents responded and signed the consent forms before the COVID-19 pandemic began. After the widespread school closures in late March 2020 across the whole US, as a response to the public health crisis, three participants (Andy, Peggy, and Charlie) continued the study through online instructions, and one participant was unable to participate due to the lack of stable internet and his parents' conflicting schedule. Note that the fourth participant never entered the intervention phase. All the names used in this dissertation are pseudonyms. The intervention schedule is presented in Appendix G. Because the second half of the intervention was online, the investigator first sent all test sheets and materials to the participants' homes in advance and then scheduled a time to pick up the forms after the intervention was completed for all three participants.

THE FIDELITY AND INTER-SCORER RELIABILITY

The fidelity results indicated that the intervention was implemented as planned based on the lesson procedure and script (Horner et al., 2005), even though it was a challenging time to complete the intervention due to the school shutdown. Two undergraduate research assistants observed the intervention using a researcher-developed checklist aligned with the scripted lessons and procedures through in-person (i.e., before the school shutdown) and online observation (i.e., after the school shutdown). On average, two out of seven lessons were checked for fidelity for each participant. The trained research assistants with education background observed the instruction delivery and assessed 28.57% of the study. Before the checking of the intervention fidelity, the assistants received training and reached interrater reliability (IRR) of 100% based on practice sessions using an audio recording of a mock lesson. Cohen's Kappa value during the training session was 1. The fidelity of the intervention implementation was 87%, a relatively high-fidelity implementation (Cicchetti, 1994; Cohen, 1960). The items that was missing the most was the ongoing feedback while practicing problems. However, this component was inevitable when the intervention switched from in-person to online. For example, the interventionist could not read the student work online while the participant was working at home. Therefore, the fidelity of the study should be perceived higher than the actual number calculated. For the inter-scorer reliability, the investigator first scored all the student answer sheets, and another doctoral-level graduate student scored the measures again. The inter-scorer agreement was 99.53%, which was highly acceptable (>90%; Neuendorf, 2002).

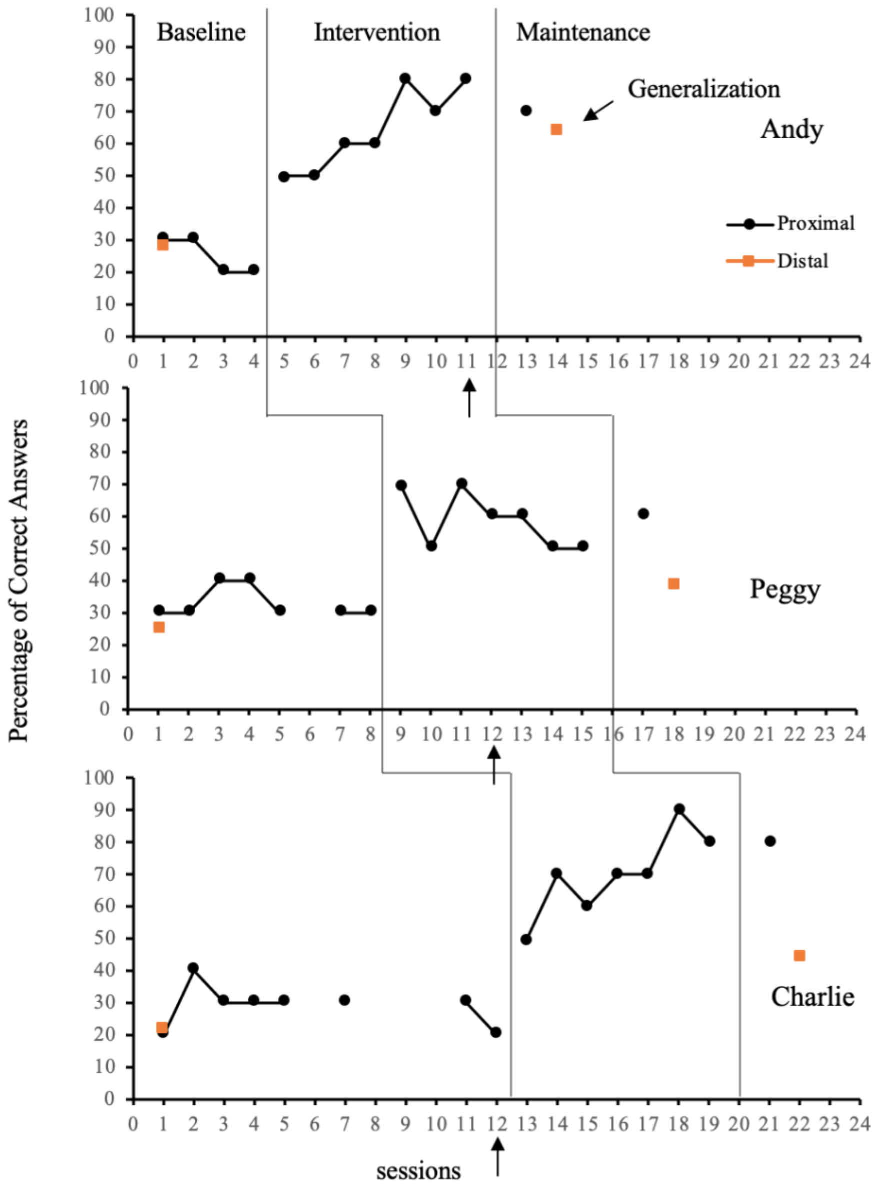
RESEARCH QUESTION 1

Research question 1 focuses on finding the immediate effects of the geometry intervention on the geometry concepts and skills measured by the proximal measure (adapted easyCBM). The lesson objectives of the geometry intervention included teaching the geometry concepts and skills required by national and state standards. For example, the students in the fourth and fifth grades should be able to identify the attributes of 2D shapes, solve perimeter and area problems, and find symmetry lines. The investigator implemented the tests with each participant throughout the intervention to assess the students' geometry performance on the targeted skills and recorded the accuracy rate of each measure.

To assess participants' performance using the adapted easyCBM proximal measure, the investigator evaluated the percentage of correct answers on the adapted easyCBM. To understand the effects of the intervention, the investigator conducted a visual analysis and computed the effect size using the NAP. Therefore, the organization of the reporting of the first research question included results of the visual analysis and the NAP.

Figure 4.1

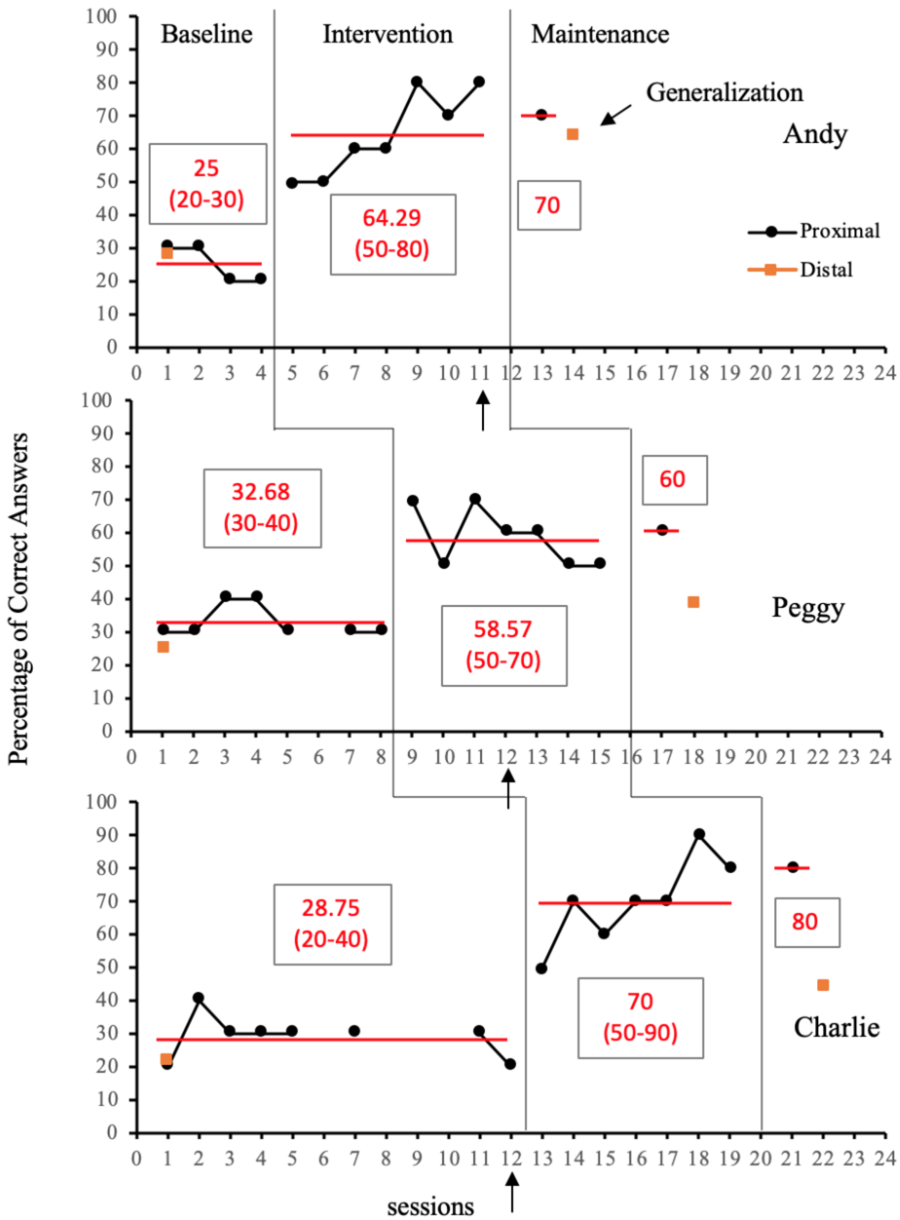
Percentage of Correct Answers on the Proximal and Distal Measures



Note. The ↑ indicated when the online intervention sessions started after the lag duration of one month.

Figure 4.2

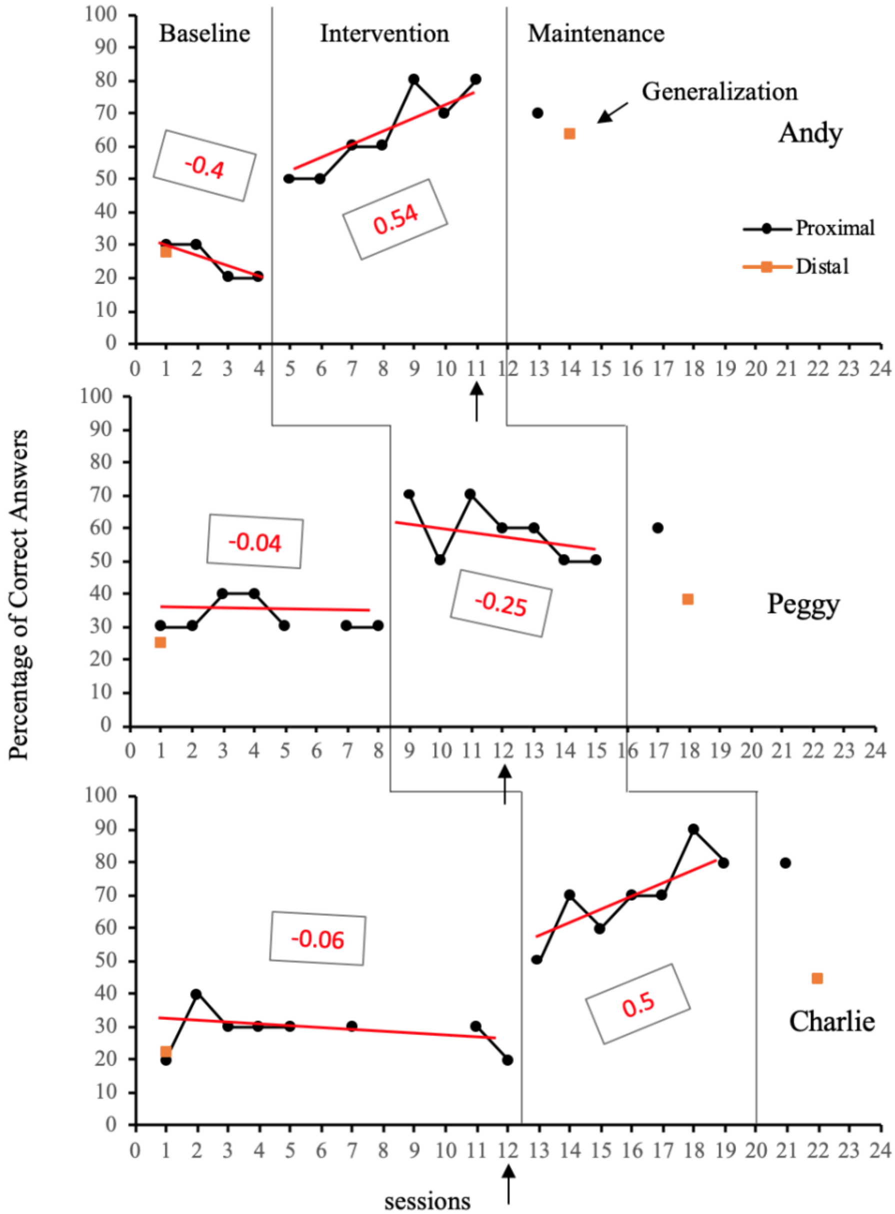
Levels for the Participant's Performances on the Proximal Measure



Note. The ↑ indicated when the online intervention sessions started after the lag duration of one month.

Figure 4.3

Trends for the Participants' Performances on the Proximal Measure



Note. The ↑ indicated when the online intervention sessions started after the lag duration of one month.

Visual Analysis

By inspecting the data, the investigator identified six features of the data points within the single case design based on the visual analysis procedures. The six aspects of visual analysis include the level, the trend, the immediacy of the effect, the variability, the overlap, and the consistency of data patterns across similar phases (Kratochwill et al., 2010). Researchers are recommended to observe the causal relationship between the geometry intervention (i.e., the IV) and the student outcomes (i.e., the DV), across three participants and for three different time points (Horner et al., 2005) to determine the existence of experimental control. In this study, the investigator analyzed the students' performances across three participants at three time points (see Figure 4.1).

Level

Figure 4.2 displays the levels of data for Andy, Peggy, and Charlie. The level refers to the mean of the data within a condition (e.g., the baseline phase). The participants' baseline levels indicate the current patterns of the students' responses, which also provide assistance for the prediction of future responses. The intervention levels show the changes in students' performances on manipulating the IV.

Andy's mean correct rates on the proximal measure (adapted easyCBM) are 25%, 64.29%, and 70% during the baseline, intervention, and maintenance phases, respectively. Andy's baseline level on adapted easyCBM is 25% (standard deviation [SD] = 0.5), and ranges from 20% to 30%. Andy showed stability the earliest among the three participants, which was after four baseline data points were collected. During the intervention phase, Andy received geometry intervention while the other two participants (i.e., Peggy and Charlie) remained in the baseline phase. Andy's level of scores during the intervention phase changed to 64.29% (SD = 1.18), and ranges from 50% to 80%; this indicates an intervention effect. The change in levels from the baseline phase to the intervention phase is 39.29%. One week after the completion of the intervention, Andy entered the maintenance phase.

Peggy was the second participant to enter the intervention phase, with means of 32.86%, 58.57%, and 60% for the baseline, intervention, and maintenance phases, respectively. Peggy's baseline level is 32.86% (SD = 0.45), and ranges from 30% to 40%. After demonstrating performance stability with seven data points during the baseline phase, Peggy started to receive geometry intervention while the last participant Charlie stayed in the baseline phase. The level of intervention for Peggy is 58.57% (SD = 0.83), and ranges from 50% to 70%. Peggy's level of change between the baseline and intervention phases is 25.71%. Peggy's maintenance level (60%) was higher than the baseline level (32.86%).

Charlie received the intervention after Andy and Peggy. Because of the disruption to the study caused by COVID-19, the investigator collected baseline data before resuming the study. The change in levels between the baseline phase and the intervention phase demonstrate an immediate intervention effect for Charlie; it went from a baseline level of 28.75% (SD = 0.60), and ranges from 20% to 40%, to an intervention level of 70% (SD = 1.20), and ranges from 50% to 90%. The level of change between the baseline and intervention phases is 41.25%. The maintenance level is also higher than the baseline level for Charlie (80%; see Figure 4.2).

Trend

The second feature of the visual analysis is the examination of the trend. The trend refers to the best-fit straight line that represents the set of data points within a condition. The investigator calculated the slopes of the data points within the baseline and intervention phases (Kratochwill et al., 2010). To analyze the trend, the investigator examined the slope and magnitude of the data points within and between each phase. The slope of data points can be upwards (positive slope), flat (zero slope), downwards (negative slope), or vertical (undefined slope; Nevison, 2014). The magnitude of the data points relates to the size of the slope, such as high, medium, and low (Kratochwill et al., 2010). The trend of each participant is displayed in Figure 4.3.

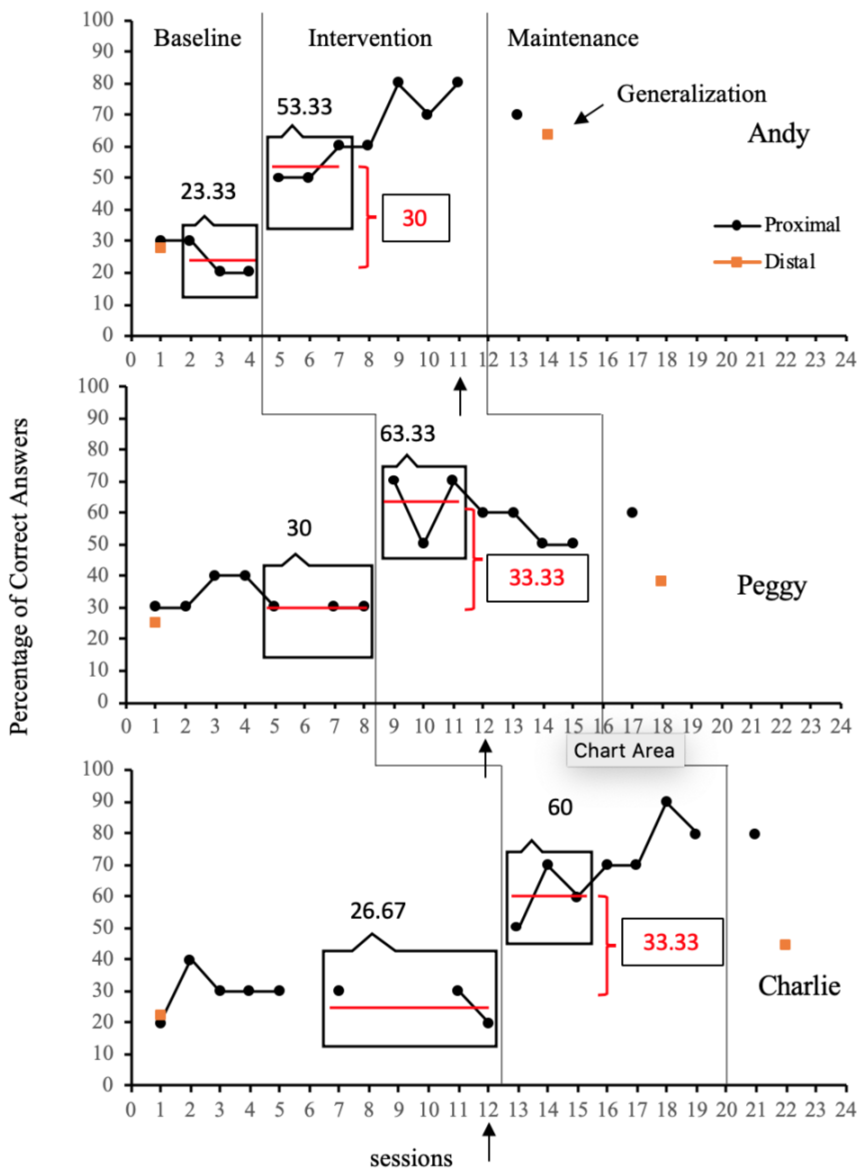
Andy's geometry performances during the baseline phase indicate a downward slope (−0.4). However, after entering the intervention phase, Andy's scores formed an upward trend on the geometry proximal measure (0.54), which reveals an increasing pattern to Andy's geometry performance after receiving the geometry intervention.

Peggy's baseline performance demonstrates a downward trend (−0.04). The investigator found a gradually decreasing pattern based on the intervention slope of −0.25, which indicates that Peggy's overall performance during the intervention phase did not improve dramatically. Note that the first three data points in the intervention phase were collected at the elementary school during in-person instruction. Starting from the fourth data point, the intervention moved to online instruction, and Peggy took the geometry tests at home, where there were some distractions. The interpretation of Peggy's performance is in Chapter 5.

With respect to Charlie's trends, the investigator has identified a decreasing pattern based on his baseline slope of −0.06. However, after receiving the intervention, Charlie's data demonstrates a firmly increasing trend during the intervention phase with a slope of 0.5. This change of trend shows that there was an intervention effect for Charlie.

Figure 4.4

Immediacy of Effect for the Participants' Performances on the Proximal Measure



Note. The ↑ indicated when the online intervention sessions started after the lag duration of one month.

Immediacy of Effect

To examine the immediacy of the effect, the investigator compared the level of the last three baseline data points to the level of the first three intervention data points for each participant (Kratochwill et al., 2010). The results of the immediacy of the effect are in Table 4.1. In general, the average immediacy of the effect for the three participants is 32.22%, with a range of 30% to 33.33%. For Andy, the immediacy of the effect was 30%, which is the difference between the two levels (23.33% and 53.33%). Peggy's and Charlie's performances reveal an immediacy of the effect of 33.3% between the baseline and intervention phases. The visualization of the immediacy of the effect is also presented in Figure 4.4.

Table 4.1

Table of the Immediacy of Effect, Variability, and Overlap

Participants	Immediacy of effect (mean & range)		Variability (SD)		Overlap (NAP)
	Baseline	Intervention	Baseline	Intervention	
Andy	23.33% (20%–30%)	53.33% (50%–60%)	0.5	1.18	No (100%)
Peggy	30.00% (30%)	63.33% (50%–70%)	0.45	0.83	No (100%)
Charlie	26.67% (20%–30%)	60.00% (50%–70%)	0.60	1.20	No (100%)
Mean	26.67%	58.89%	0.52	1.07	NA (100%)

Variability

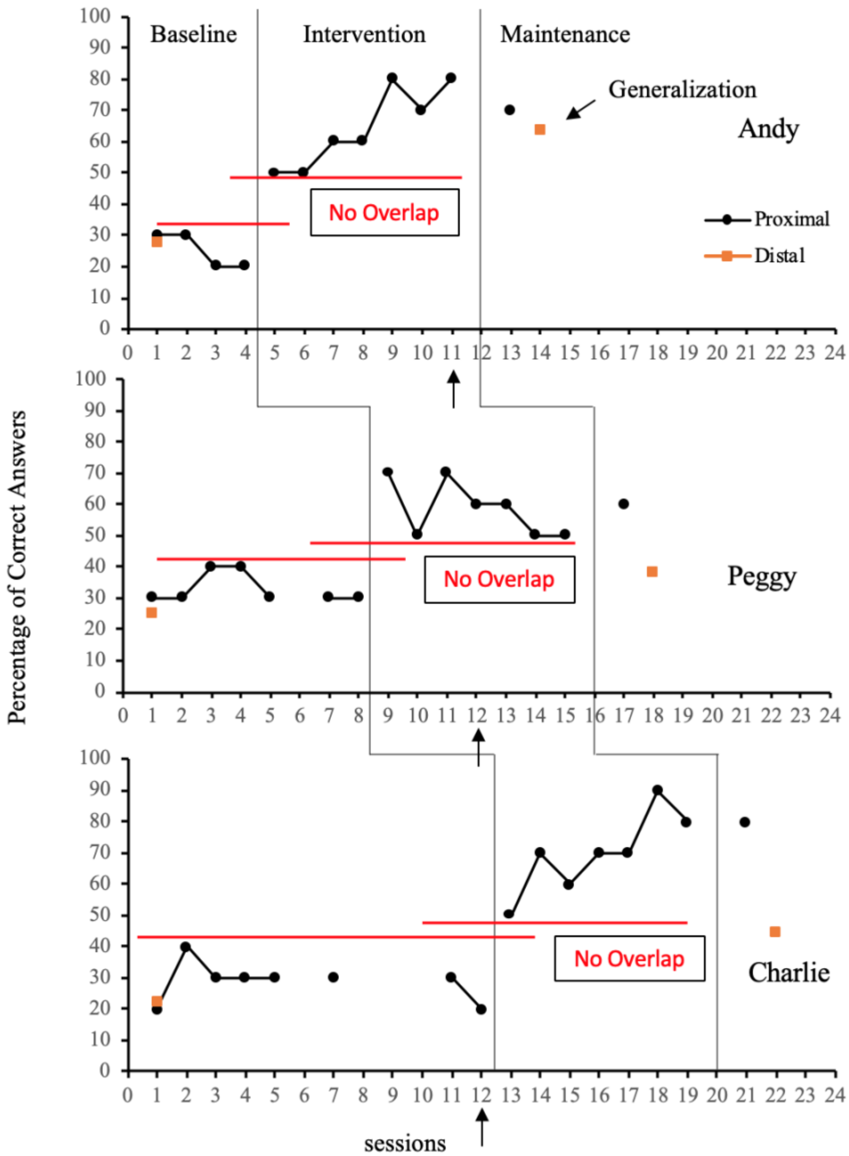
The results regarding the variability of the students' performances are presented in Table 4.2. The variability refers to the degree to which each data point deviates from the overall trend (Kratochwill et al., 2010). The investigator used the SD and range to evaluate this feature of visual analysis. The SD of Andy's data points during the baseline phase is 0.5, with a range of 20% to 30%. The SD for the intervention phase is 1.18, with a range of 50% to 80%. For Peggy, the SD of the variabilities of the results of the baseline and intervention phases are 0.45 (with a range of 30% to 40%) and 0.60 (with a range of 50% to 70%).

Overlap

The overlap of data points is the percentage of data from the baseline phase that overlaps with the data in the intervention phase, which confirms there has been an intervention effect (Kratochwill et al., 2010). The investigator examined the overlap across the three participants by comparing the highest data point during the baseline and the lowest data point in the intervention phase (see Figure 4.5). No participants have overlapping data points between the baseline phase and the intervention phase. For example, the highest percentage-correct rate for Andy during the baseline phase is 30%, and the lowest one during the intervention phase is 50%.

Figure 4.5

Overlap Data Points for the Participant's Performance on the Proximal Measure



Note. The ↑ indicated when the online intervention sessions started after the lag duration of one month.

Consistency of Data Patterns

Determining the consistency of the data patterns involves examining the data patterns across similar phases and for all participants (Kratochwill et al., 2010). The investigator observed and compared the data within and between phases (baseline and intervention phases) to find predictive patterns for the intervention-outcome variable (i.e., students' geometry performances).

For Andy, the amount of variation in the baseline phase is relatively small (SD = 0.5, range = 20%–30%) and it is significantly high in the intervention phase (SD = 1.18, range = 50%–80%). A consistent pattern is seen with Charlie. The variation in the data points for Charlie is similar to Andy's. Charlie has the largest variation in data points in both the baseline phase (SD = 0.60, range = 20%–40%) and the intervention phase (SD = 1.20, range = 50%–90%). Peggy's baseline data indicates a pattern that is consistent with the other two participants, even though the variation is the smallest (SD = 0.45, range = 30%–40%) for Peggy. Peggy's intervention score has a relatively small variation (SD = 0.83, range = 50%–70%) compared with the other participants. The interpretation of the results is in the next chapter (Discussion).

Effect Sizes: Proximal Measure

To investigate the immediate effect of the geometry intervention, the investigator also calculated the effect sizes of the following scale using the NAP, which is recommended for use in examining a single case design (Parker & Vannest, 2009). To determine the NAP, the investigator calculated both the total number of possible pairs of data points between the baseline and intervention phases and the total number of pairs of non-overlapping data. Next, the percentage of non-overlap was calculated by dividing the total number of pairs of data points with the total number of pairs of non-overlap data multiplied by 100. Because there are no overlapping data points across all participants (see Figure 4.5), the NAPs for Andy, Peggy and Charlie are all 100% (see Table 4.1 under NAP).

Summary

In summary, to answer the first research question (what is the immediate effect of a geometry intervention on the geometry performances of fourth- and fifth-grade students with LD, as measured by a proximal measure [adapted easyCBM]?), the investigator examined the data using visual analysis and by calculating the effect sizes.

For the visual analysis, in terms of the levels of data, all participants demonstrated an increased level of performance from the baseline phase to the intervention phase. Charlie demonstrated the highest level of change between the baseline and intervention phases among the three participants (41.25%), while Peggy showed the smallest change between the two phases (25.7%, see Figure 4.1). There was an increased level of change between the intervention and maintenance phases for all participants, which means the level of maintenance was higher than the level of intervention. For example, Charlie's maintenance level increased to 80% from 70% in the intervention phase.

Regarding the trend of the data, even though there was a consistently positive change in the levels of student performances across the three participants, not all participants demonstrated an upward trend after receiving the intervention. By assessing the trend in the data, the investigator found that the three participants reached a stable baseline, but only two participants (Andy and Charlie) show an upward trend after the introduction of the intervention. Even though Peggy's data points reveal a downward trend (−0.25), his performance during the intervention phase is above the baseline phase's highest data point and his performance remained higher during the maintenance phase. It is worth noting that Peggy's first three data points in the intervention phase were collected when receiving instruction in person; however, the rest of the data points were collected after the instruction moved online.

The immediacy of the effect was calculated across the three participants with an average increase of 32.3% between the baseline phase (range = 23.3%–26.7%) and the intervention phase (range = 53.3%–63.3%). An overlap between the baseline and intervention phases did not occur among the three participants, which indicates a higher level of performance in the intervention phase than that of baseline. Regarding the variability, the mean variability (i.e., the SD) in the baseline phase (SD = 0.52, range = 0.45–0.60) is smaller than that in the intervention phase (SD = 1.07, range = 0.83–1.2) for all participants.

For the effect sizes, the intervention probes for Andy, Peggy, and Charlie were above the baseline probes. There were no overlap data points between baseline and intervention phases across all participants (NAP = 100%).

RESEARCH QUESTION 2

Research question 2 focuses on finding the maintenance effects of the geometry intervention on the geometry concepts and skills measured by the proximal measure (adapted easyCBM). The investigator collected data on the fourth-grade adapted easyCBM to investigate the degree to which the students maintained the geometry knowledge they had learned during the intervention at a point one week after the intervention. The results for this research question can provide potential evidence regarding the long-term maintenance effect on geometry outcomes of the students.

All participants maintained a higher performance level during the maintenance phase compared to previous phases (i.e., the baseline and intervention phases). There is a consistent increase in the maintenance level compared to that of the intervention and baseline phases. For example, Andy entered the maintenance phase first after the intervention. The level of Andy's maintenance data (70%) exceeds the level of baseline (25%) and intervention (64.29%) data, which demonstrates that Andy's learned geometry concepts and skills remained one week after the completion of the intervention (after the instruction and feedback were removed; see Figure 4.2). Peggy's maintenance level is 60%, which is slightly above the intervention level (58.57%). Charlie's maintenance performance also remained at a high level (80%) compared to the baseline (28.75%) and intervention (70%) levels.

RESEARCH QUESTION 3

To answer research question 3 (to what extent do the fourth- and fifth-grade students with LD generalize their geometry knowledge to a distal measure [KeyMath-3 geometry subtest]?), the investigator implemented both a pretest and a post-test using a KeyMath-3 geometry subtest and then examined the results. Before starting the geometry intervention in the early spring semester of 2020, all participants completed the pretest. The post-tests were in the middle May, 2020. The tests were implemented individually to each participant. Each student answered questions verbally or chose a correct answer among several choices provided on the KeyMath-3 assessment easel after hearing the question from the test administrator (the investigator). There are 36 items in the geometry subtest. Once a student provides five incorrect answers consecutively, the test stops. The investigator writes down the raw score by subtracting the number of incorrect answers from the test item at which the participant stops. For example, if the student stops at item 15 and he/she has answered seven questions incorrectly, the raw score will be eight.

The investigator checked the results against the KeyMath-3 test manual; these generalization results are reported in Table 4.2. The interpretation of the scale scores is based on Table 3.4. Andy's pretest and post-test raw scores on the KeyMath-3 geometry subtest are 10 and 23. Andy's pretest scale score is 5, which is "well-below average," and the post-test scale score is 10, which indicates an "average" level (see Tables 3.4 and 4.2). Andy's total correct rate for the KeyMath-3 geometry subtest increased from 41.67% to 63.89% (see Figure 4.1).

Peggy's raw score increased by one point from before to after the intervention. His pretest raw score is 9 and the post-test raw score is 10. Peggy's pretest and post-test scale scores are both 4, which is classed as "well-below average" before and after the intervention. There are many possible reasons for his performance, which will be interpreted in the next chapter.

Table 4.2

Pretest and Post-test Results for the KeyMath-3 Geometry Subtest

Variable		Raw score (% correct)	Scale score	Descriptive interpretation
Andy	Pretest	10 (27.78)	4	Well-below average
	Post-test	23 (63.89)	10	Average
Peggy	Pretest	9 (25)	4	Well-below average
	Post-test	10 (38.46)	4	Well-below average
Charlie	Pretest	8 (22.22)	3	Well-below average
	Post-test	16 (44.44)	8	Average

Charlie's pretest score was the lowest, which is 8. The corresponding pretest scale score is "well-below average." However, after receiving the intervention, Charlie's post-test score increased to 16, which brought him up to the level of "average." This result indicates Charlie's ability to generalize the geometry concepts and skills from the geometry intervention to other geometry questions. Also, Charlie increased the correct rate from 22.22% to 44.44%.

RESEARCH QUESTION 4

The purpose of research question 4 is to examine the participants' perspectives on the geometry intervention. To answer this research question, the investigator created a social validity form and asked the participants to complete it one week after the intervention finished. All participants completed the items at home alone or in the presence of their parents'.

The summary of the results of each question on the social validity form is in Table 4.3. The social validity form was used to measure the students' perceptions of the geometry subject and the ICs of the geometry intervention. The form consisted of nine items, using a five-point Likert scale: strongly disagree = 1, somewhat disagree = 2, do not agree or disagree = 3, somewhat agree = 4, and strongly agree = 5. The investigator developed three categories of questions: (a) general perspectives on geometry; (b) geometry effect on geometry problem-solving; and (c) intervention components. Questions 1 and 2 involve the examination of the student's general perspective toward geometry after the geometry intervention. Questions 3 and 7 are related to the student's perspectives on the geometry intervention effect. Questions 4, 5, 6. 8, and 9 evaluate the intervention components, such as the Frayer model, and the concrete and semi-concrete manipulatives.

In general, the results of the social validity survey reflect positive perceptions of most of the items on the social validity form, and the range of the rating scale for all items is between 3.67 and 4.67. The students' ratings on seven items are over 4. The highest rating item is question 4, which relates to the use of the Frayer model in teaching geometry vocabulary (rating = 4.67). The Frayer model activity included in each lesson involved writing and drawing geometric shapes. Interestingly, even though all students spent a long time creating the model and struggled with writing the shapes' definitions and attributes, they seemed to enjoy the experience. The lowest ratings are related to the general perception of geometry (rating = 3.67). Two participants either strongly or somewhat agreed on the statements that they like geometry and that it is important to learn geometry; however, one participant somewhat disagreed on both of the statements. The ratings of Andy and Charlie are relatively higher than those of Peggy, who displayed some behavior issues during the intervention (e.g., tantrums and anxiety).

However, some parts need improvement. For example, one participant thought that the flash cards did not help with the shape concept memorization even though he seemed to like the flashcard activity during the session. Moreover, he also did not like geometry or think geometry is important (rating = 2). Therefore, the investigator can explore other teaching methods besides the use of flashcards when review geometry content in the future.

Table 4.3

Students Perspectives on the Geometry Intervention

Social validity questions	Andy	Peggy	Charlie	Average rating
1. I like geometry.	5	2	4	3.67
2. I think geometry is important.	4	2	5	3.67
3. I know more about shapes and shape attributes to solve geometry problems after the instruction.	4	4	4	4.00
4. The Frayer model helps me remembering the knowledge related to shapes.	5	4	5	4.67
5. The geometry words we learned helped me do better in geometry.	5	3	5	4.33
6. Using different materials with shapes made geometry easier to understand.	4	4	5	4.33
7. I feel as though I was able to finish many of the problems independently on the worksheets.	4	4	4	4.00
8. The flashcards model help me remembering the knowledge related to shapes.	5	2	5	4.00
9. I think that talking about how to solve a problem helps me understand the problem better.	5	4	4	4.33

In summary, the intervention is viewed as being beneficial and effective for most participants. Two participants had positive perspectives on geometry in general; however, one student reported his dislike of geometry. In regard to the students' perspectives on the intervention effect, all participants somewhat agreed that they had improved their geometry skills after the intervention. In terms of the geometry intervention, all participants strongly or somewhat believed that the key components were beneficial. These components include the teaching of geometry vocabulary using the Frayer model, the use of different geometry learning materials, the use of flash cards to review key concepts, and the promotion of students' verbalization during the instruction. Even though the intervention was disrupted in the middle and resumed online later, the participants believed the whole learning experience was beneficial.

SUMMARY OF THE CHAPTER

The purpose of this study is to investigate the effects of a geometry intervention on standards-aligned geometry concepts and skills for three elementary students (fourth and fifth grades). This chapter has reported the results related to the four research questions for the geometry intervention. Overall, through analyzing the results, both the proximal and distal measures show the positive effects of the geometry intervention on improving students' geometry performances. The findings of the visual analysis and the calculation of the effect sizes indicate a functional relation between the geometry intervention and the students' geometry performance.

The visual analysis of the geometry data points from the proximal measure (fourth-grade adapted easyCBM geometry) for research question 1 demonstrate a positive effect in improving students' geometry performances. Regarding the performance levels for all participants, there were low levels of geometry performance in the proximal measures observed during the baseline phase and higher levels of geometry performance during the intervention phase. The average increase in level between the baseline and intervention phases across three participants is 35.42%. Regarding the trends, the results indicate that the three participants maintained a stable baseline before the intervention, and then all participants increased their geometry performances sequentially. In terms of the immediacy of the effect, the average increase in the students' performance between the baseline and intervention phases is 32.2% across the three participants. Regarding the variability, the results show a relatively stable variability in the baseline phase (mean SD = 0.52, range = 0.45–0.60) and an increased variability in the intervention phase (mean SD = 1.07, range = 0.83–1.20) across all participants. Regarding the overlap of data points, all participants have no overlapping data points between the baseline and intervention phases.

The trend of one participant (Peggy) did not go upwards in the intervention phase after the online teaching started, however, Peggy's increased levels of performance in both intervention and maintenance phases and the results of immediacy of effect showed that there is a functional relation between the independent variable (the geometry intervention) and the dependent variable (the participants' geometry performances).

In addition to the visual analysis, the investigator also computed the effect sizes of the geometry data points on the proximal measure. The results of the NAP for the three participants were all 100%, which indicates that there is no overlap in data points between the baseline phase and intervention phase, and there is a strong effect size.

The investigator also examined the maintenance and generalization effect of the geometry intervention corresponding to research questions 2 and 3. Research question 2 evaluates the maintenance effect of the geometry intervention after the investigator implemented the instructional procedures with fidelity. The results of the maintenance tests demonstrate a strong intervention effect as all students maintained a high level of performance one week after the intervention. The performance levels during the maintenance phase were higher than for the baseline and intervention phases for Andy, Peggy, and Charlie. Research question 3 involves examining the generalizability of the geometry problem-solving skills using the distal measure (Keymath-3 geometry subtest). The investigator compared the pretest and post-test scores on Keymath-3 geometry for each participant and found that two participants were able to generalize their problem-solving improvement on the geometry questions outside the geometry intervention. Before the intervention, Andy and Charlie were categorized as being of "well-below average" level in their geometry performances. However, after receiving the intervention, both increased their levels to "average."

The purpose of research question 4 is to understand the participants' learning experience of the geometry intervention. The investigator developed a social validity form to investigate the participants' opinions on geometry in general, geometry problem-solving skills, and the intervention components. The answers reflect positive perceptions of most items across all participants, with some exceptions for Peggy.

Chapter 5: Discussion

The purpose of the study is to examine the effect of the geometry intervention on the geometry performances of fourth and fifth-grade students with LD. In addition, the investigator explored how the skills learned from the geometry intervention can be maintained for a longer period (one week after the instruction).

Elementary geometry concepts and skills are required knowledge for K-12 students, according to the National Council of Teachers of Mathematics (2000). Researchers have tried to understand how students learn geometry, and they have facilitated the development of the geometry curriculum and instructions. For example, van Hiele-Gedolf (1957) proposes a five-level geometric-thinking model, which describes how students' understanding of geometry proceeds sequentially from the lower level of geometry thought to the higher ones. Some researchers propose that geometry is learned through different representational modes (e.g., concrete modeling, pictorial modeling, and symbolic representations; Lesh, 1978). These theories and models have informed teachers that they should provide learning opportunities with different representations to improve elementary students' understanding of geometry concepts (van de Walle, 2004).

Students with LD) often have difficulty with basic mathematics concepts and procedural knowledge (Geary, 2004; National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). National and international data shows the low performances of students with disabilities, including students with learning disabilities in the U.S. (e.g., NAEP, 2019; TISSM, 2015). However, research findings demonstrate that students with disabilities improved their geometry outcomes after receiving supplemental geometry instruction (Bergstrom & Zhang, 2016). There is also evidence that students with LD) are able to increase their geometry problem-solving skills, even though most studies were conducted with secondary-level participants with LD. A very limited number of research projects have included elementary students with LD (Liu et al., in press).

Therefore, the investigator conducted this study with students with LD at elementary level and examined the effects of the geometry intervention. In the first section of this chapter, there is an interpretation of the results based on each research question and how the findings from the study relate to the current literature. Next, the limitations and future research are discussed. Finally, the implications for research and practice are provided.

DISCUSSION OF RESULTS

During the intervention implementation process, COVID-19 interrupted the proposed intervention. The investigator determined to complete the study, and the data showed positive findings as indicated in Chapter 4. Due to COVID-19, the first half of the intervention was delivered in person, and the second half was online. Specifically, the investigator had to change the intervention settings from in-person at school to remote learning at home; and revised the intervention schedule after a one-month lag, and the lesson plans designed for in-person instruction had to be delivered online. The data collection was at students' homes with the help of parents.

There are many difficulties in resuming the intervention online. During the one-month gap time, the investigator tried to communicate with the school principal and teachers. It is a special time due to COVID-19, and all school staffs were busy under new circumstances. The investigator had to wait for three weeks to figure out the new intervention schedule for the participants. There are also variations among online learning participants, e.g., home environment and the student engagement level. The investigator discussed this in the Limitation Section.

Some parents worked at home all the time while some parents had to work outside. That was also one reason that the fourth participant ended up dropping the study (i.e., conflict schedule). It was challenging to recruit another participant at that time, so in total, three participants completed the study. To minimize the risk and collect data needed for the study, the investigator prepared an intervention package for each participant and placed it at the door of each participant's home. After the investigator left, the parents opened the door and picked up the package with learning materials, including intervention practice sheets, test sheets, scratch paper, pencils, and erasers. The investigator picked up the materials from their homes once the intervention was completed.

This study was a multiple probe single case study across three participants. The participants' responses on proximal (fourth-grade adapted easyCBM) and distal measures (KeyMath-3 Geometry subtest), and social validity forms provided the quantitative data to answer the following research questions:

1. What is the immediate effect of a geometry intervention on the geometry performances of fourth- and fifth-grade students with LD as measured by a proximal measure (adapted easyCBM)?
2. To what extent do the fourth- and fifth-grade students with LD maintain their geometry performance one week after the conclusion of the intervention as measured by a proximal measure (adapted easyCBM)?
3. To what extent do the fourth- and fifth-grade students with LD generalize their geometry knowledge to a distal measure (KeyMath-3 geometry subtest)?
4. What are the perspectives of the fourth- and fifth-grade students with LD on the geometry intervention?

Unlike the previous research, which mostly involved solving perimeter and area problems, the participants in this study also demonstrated the acquisition of skills for foundational concepts, such as basic geometry concepts (e.g., parallel lines and perpendicular lines), recognition of 2D-shape attributes (e.g., sides and vertices), and recognition of symmetry lines. Overall, the participants in this study demonstrated improvements in geometry after receiving the geometry intervention, which is consistent with the previous research on teaching mathematics to elementary-level students with mathematics difficulties (Bryant et al., 2011; Bryant et al., 2014). Additionally, all three participants maintained their knowledge with a high correct rate one week after the intervention, and two participants transferred their skills to a variety of geometry problems, as revealed through the distal measure (Keymath-3 Geometry subtest). Two participants had an overall high rating for the geometry intervention based on the results of the social validity form, which concerned their overall learning experience.

Research Question 1

The first research question was used to examine the immediate effect of the geometry intervention. The effectiveness of the intervention was evident in the changes in the probes for each participant, when examined using visual analysis and the effect-size calculation. During the screening process, all participants scored below the 25th percentile on the third-grade easyCBM geometry pretest. During the baseline phase, the participants' total average percentage of correct answers (i.e., the average of Andy, Peggy, and Charlie) was 28.87%. However, the probes during the intervention indicated an increased mastery of geometry concepts, with an overall average of 64.29%. The positive change revealed the immediate effect of the intervention, which also showed a functional relation between the geometry intervention and the geometry performances.

The results of the present geometry intervention are also aligned with the previous research in many aspects. For example, the three elementary students with LD in this study were able to learn geometry, and they improved their geometry concepts and skills after receiving the intervention, which was suggested in the previous research (Xin & Hord, 2013). As indicated in Chapter 4, all participants improved their geometry performances. Several possible explanations are discussed in the following.

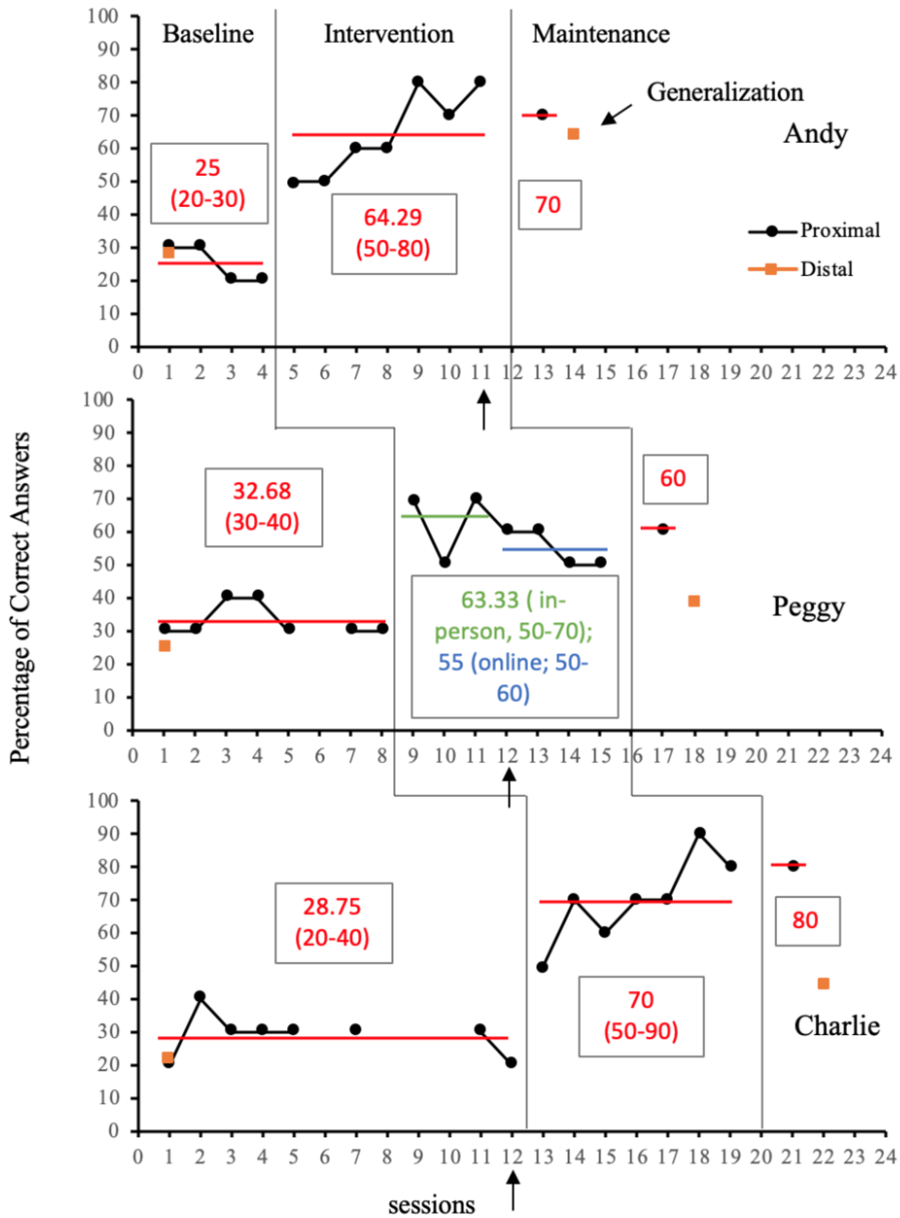
One explanation of the participants' geometry improvements may be related to their better acquisition of geometry vocabulary. The geometry intervention included two intervention components to teach geometry vocabulary: the use of the Frayer model and the flashcards. The Fray model provided participants with opportunities to gain a deeper understanding of the geometry vocabulary through writing the definitions and drawing the examples and non-examples (Frayer et al., 1969). The use of flash cards was also an engaging activity used to review and memorize the pictorial representations of shapes and the attributes of shapes (Komachali & Khodareza, 2012). The participants were excited about how many more words/phrases they recognized at each session, and they became motivated when they knew more geometry vocabulary.

Second, the design of the geometry intervention might also play an important role in the participants' improved geometry performances. Based on the literature, the investigator incorporated several effective ICs into the study. The research confirmed that some ICs (e.g., instructional sequencing, control of difficulty, multiple representations, and one-on-one instruction) helped improve the mathematics performances of students with disabilities or with LD (Doabler et al., 2019; Gersten et al., 2009; Kozulin & Kazaz, 2016; Maccini & Hughes, 2000; Satsangi, Hammer, & Bouck, 2019; Strickland & Maccini, 2012). The results of the present study were consistent with the finding that modeling, prompting, and independent practices helped secondary students with LD learn perimeter and area problem-solving skills (Cass et al., 2003). The use of multiple representations, including concrete and virtual models, is also recommended (Bouck et al., 2015; Xin & Hord, 2013). The investigator used concrete (e.g., geoboard, pattern blocks, and AngLegs) and representational (e.g., pictures) models to scaffold the geometry concepts and elaborated on the key features of each geometry topic. For example, when teaching parallel lines, the investigator first modeled what parallel lines were using concrete models with a geoboard, and then asked the participant to find the parallel lines in the classroom / their home, draw the lines on the notebook, and explain the understanding of parallel lines.

Third, despite the change of time and settings, the results showed consistency patterns of students' geometry performances, which revealed the intervention's robustness and strength. The effect of time and settings are controlled within the increased patterns of the data in this study. The continuity of the intervention sessions provided participants with a focused learning time for concentrating on and making a consistent effort toward the targeted skills. For example, the first participant, Andy, completed the majority of the geometry intervention (from lesson 1 to lesson 6) without an interruption before the spring break. Most of her intervention sessions were in person in the elementary classroom. The third participant, Charlie, also received his instructions consistently remotely after the school shutdown due to COVID-19. However, Peggy had to stop receiving in-person instruction after lesson 3, and resumed the intervention at home, completing lesson 4 to lesson 7 online after one-month gap. Even though there was an inconsistency of time and setting in the intervention phase, as indicated in Figure 5.1, Peggy's performances in the intervention (58.57%) and maintenance phases remained at a high level before and after the online instruction (63.33%; 55%; Figure 5.1), which indicated an intervention effect. This improvement of performances is important given the meaningfulness of the geometry topics at elementary grades (e.g., the geometry attribute knowledge lays a foundation for more advanced geometry at secondary level).

Figure 5.1

Peggy's Levels of Performance Before and After the Interruption



Note. The ↑ indicated when the online intervention sessions started after the lag duration of one month.

Fourth, the online teaching might have influenced the intervention delivery compared to the in-person teaching. There are benefits of online teaching, and according to an annual review by Watson et al. (2013) analyzing on national data, the main subject areas of online course activities are language arts (23.1%), mathematics (22.7%), science (14.1%), and social studies (14%). Other online courses (21.61%) were related to health, world languages, or arts. Online or blended instruction can be an option for meeting various learning needs for K-12 students, including struggling learners (Smith & Basham, 2014). For example, online learning allows flexibility in the time and location of receiving instruction. However, challenges exist in an online teaching environment, especially for students with disabilities, based on their individual learning needs (Greer et al., 2014). For example, the online teaching format met Peggy's content-learning needs, but it did not match his learning style. He was not very motivated by online instruction during the intervention sessions. Considering his home environment (e.g., the noise level and many distractions), online learning might not be the best choice for some students.

Fifth, the home environment of each participant differed, which was the same as the participant engagement level. Andy and Charlie had a relatively quiet home environment while receiving the instruction at home. However, Peggy was often distracted because his home environment was rather noisy and crowded, with two younger siblings and multiple adults at home. The investigator tried to persuade Peggy to go to a quiet room/place, and his parents also tried to let Peggy concentrate, but it did not work well.

Peggy was a very active student and used to have many behavioral issues at school, while the other two participants were collaborative and enjoyed the intervention sessions. Peggy was also sensitive. For example, whenever he answered some questions correctly, he was motivated and tended to do more. However, when he thought he did a bad job, he got upset and gave up easily. The investigator tried ways to motivate Peggy; for example, praising him verbally or providing him with opportunities to work with concrete geometry models. During the online intervention sessions, Peggy sometimes ran away from the camera, which made it hard to continue the session without a break. However, the other two participants were able to maintain a stable level of attention throughout the instructional time.

Research Question 2

The second research question explored the extent to which the elementary students with LD maintain their geometry performance one week after the geometry intervention had completed. The participants demonstrated a high degree of maintenance with an average level of correct answers of 70% (range = 60%–80%), which is consistent with the previous research with secondary school participants (Cihak & Bowlin, 2009; Strickland & Maccini, 2012). This is a good outcome because having a poor retention of skills is considered to be a characteristic of students with LD (Bley & Thornton, 2001; Geary, 2004). In addition, Andy's maintenance level (70%) was higher than Peggy's (60%) and lower than Charlie's (80%). The maintenance levels of all participants were higher than those of the intervention phases. For example, Andy's maintenance level was

70% compared to the intervention level of 64.29%, even though there were three intervention probes that were higher than the maintenance level (see Figure 4.2).

The item analysis revealed that participants had trouble recognizing different types of angles. For example, Peggy and Charlie selected the wrong answers when identifying acute and right angles. It was partly because the angle recognition was introduced early in the lessons, and the participant might need more opportunities and time to review and practice before moving to the maintenance phase. Additionally, Andy responded incorrectly to questions related to parallel lines and the identification of symmetry lines, although she had answered similar questions correctly before. It is also possible that the participants' level of attention and engagement was lower, and they might have chosen an answer without reading the question carefully at the time of testing.

Research Question 3

The third research question examined the how students with LD transferred the geometry concepts and skills learned from the intervention to other geometry problems. Transferring mathematical concepts and skills to novel situations can be difficult for students with LD and typical students (Bley & Thornton, 2001; Fuchs & Fuchs, 2007). Two participants showed an increased range across geometry problem types in the generalization measures, which is consistent with previous research (Maccini & Hughes, 2000; Strickland & Maccini, 2012; Satsangi & Bouck, 2015).

Andy's better performance as given by the distal measure demonstrated that she could transfer the skills from the geometry intervention to other geometry problems.

Specifically, Andy increased the raw score from 10 to 23, with 13 points more from the questions responded to correctly at the Keymath-3 geometry subtest post-test. The items that Andy had previously answered incorrectly reflect that her skills acquired from the geometry intervention were transferrable. The concepts and skills that Andy generalized included (a) identifying objects based on clues of position or location related to others pictured; (b) predicting the resulting views of a given object (a cube) from different views, or by adding or removing some parts; (c) identifying the cross-section formed by slicing through a given solid at an angle; (d) identifying shapes that were rotated and flipped; (e) identifying the similarities and differences between shapes; and (f) identifying parallel lines or perpendicular lines in pictures of real-world objects.

On the contrary, Peggy had answered only one more question correctly from pretest to post-test. The item Peggy answered correctly at post-test was related to the shape-transformation ability; for example, selecting the shape that was rotated and flipped based on the original one. However, Charlie generalized more skills after the intervention: (a) applying the terms right and left; (b) identifying different views of the same object; (c) identifying the resulting view of a simple cube structure if cubes were removed or added; (d) identifying the shape that does not belong in a given set of shapes, by shape attributes, and giving reasons; (e) determining the number of corners and faces of a prism; and (f) comparing angles through measurement. These results indicated that the acquisition of 2D geometry concepts and skills were beneficial for the participants solving 3D problems; for example, the spatial understanding of cubes and prisms.

The study's findings indicate that the elementary students with LD were able to master the geometry objectives in a relatively short period with the ability to generalize the skills they acquired from the intervention to more complex (near- or far-transfer) geometry problems. The participants in this study acquired new knowledge and transferred skills to other situations after the investigator provided a supportive environment with novel examples and explanations to the new situation. For example, the participants might first have recognized the perpendicular lines within right triangles by visualizing pictures of the shapes and identifying the unique attributes (e.g., a right angle within the shape). Then, they found similar rules across different shapes or properties. In making sense of a new situation, one could process the new information through either assimilation or accommodation (Piaget, 1964). Assimilation means that someone acquires new knowledge based on their previous cognitive framework. For example, students might have previously perceived that all triangles have three sides and three angles, but they add the new examples of equilateral triangles or right triangles to their existing pool of information by knowing that triangles can come in different types and sizes. Accommodation implies that one creates a new cognitive framework because the new situation did not fit into the previous framework. In sum, the evidence of generalizability of the skills was found in this study.

Research Question 4

The fourth research question investigated the participants' perceptions of the geometry intervention. The participants' feedback after the intervention was important because this information helps researchers understand the cause of the intervention effect and develop viable programs that benefit more participants (Carr et al., 1999). The responses were mixed when the participants were asked about their perceptions of geometry. One reason for the relatively lower rating of one participant (Peggy) was that the participant did not enjoy the one-on-one learning format. Specifically, the participant expressed his strong preference for learning knowledge together with other students instead of being alone with the teacher. Possibly, he had previously had negative experiences with one-on-one instruction.

Two participants (Andy and Charlie) demonstrated that they liked geometry and they believed geometry was important. Additionally, all the participants thought their knowledge about geometry had increased after the intervention and felt confident about solving many geometry problems independently. All participants believed that the use of the Frayer model and flashcards were helpful in learning geometry vocabulary, which was also beneficial for solving geometry problems. All participants provided positive feedback on the use of multiple representations (e.g., concrete and pictorial manipulatives) during the intervention. Thus, the use of the social validity form in the present study was aligned with the practices that were recommended by the previous research (Lindo & Elleman, 2010).

LIMITATIONS AND FUTURE RESEARCH

This geometry intervention was a single-case design with an experimental control of the independent variable (i.e., the geometry intervention). The investigator found evidence of a functional relation between the independent variable and dependent variable across three participants with LD at the elementary level. The overall performances of all participants indicated an intervention effect. Despite the promising findings, this study had limitations on three aspects: (a) the validity of the intervention, (b) the multiple-component design; and (c) the maintenance probe.

First, there was no content validity of the geometry intervention and adapted easyCBM measure. Content validity of the geometry intervention represents the degree to which the instructional materials (i.e., geometry intervention lesson plan) represent the geometry concepts and skills required by the national or state level. Even though the lessons were developed after the investigator reviewing national and state math standards under the supervision of the investigator's supervisor, no data was available to ensure the content or construct validity of this intervention. The lack of validity in the adapted easyCBM might also limit the ability to make inferences from the test results. Additional evidence is needed in future research to provide validity information.

Second, the present study was a multi-component study with different ICs and strategies based on previous research (e.g., Gagnon & Maccini, 2005). However, the differences in the effects of the intervention components were unclear. Therefore, future research should assess the effects of different ICs using different design types, such as group designs. Additional study that systematically manipulatives specific IC(s) is recommended.

Third, only one short-term maintenance probe was collected after the intervention. Due to the very strict timeline before the spring semester ended, the investigator had to implement the maintenance test one week after the intervention. However, in the future, studies replicated with multiple maintenance probes and at different time points are needed to reach a conclusion regarding the long-term maintenance effects.

From the future intervention development perspective, it is suggested that future researchers can design interventions that can switch between in-person and remote learning. For the current geometry intervention, because of the sudden change of instructional format, the investigator had to pause the intervention for a month and resumed the study using synchronous teaching for the study consistency across three participants. The impact of switching to remote learning on the original design of the intervention is unknown. Synchronous learning required the teaching and learning needed to happen at the same time, which can limit student learning. Instead, there are advantages of asynchronous learning or hybrid learning, because the learning can be more flexible and occurs at any time or any place as long as the educational channel is available. Future research should design a study that can accommodate the change of intervention settings and intervention time.

Another suggestion is that more research is needed to provide additional evidence to facilitate understanding the teaching of geometry. In this study, there were three participants with learning disabilities in math. However, even though these participants are heterogeneous, for example, Andy was diagnosed with dyslexia, Peggy had an IEP goal in written expression, and Charlie has an IEP goal in math calculation, the results of the study indicated a strong intervention effect across participants. Future research can provide more evidence for external validity using replication studies across different participants or focusing only on participants with a more focused disability type.

IMPLICATIONS TO PRACTICE

The findings of this study provide several implications for practice. First, online or remote delivery can be the “new normal.” Under the new circumstances due to COVID-19, American education has never experienced such constraints and interruptions of educational research (DeMatthews et al., 2020). How to conduct and continue educational research and how to allocate resources efficiently are essential topics to discuss. Even though the priority was given to issues, such as responding to the risk of infection or minimizing the impact on the economy currently, as researchers, we need to actively work on how to continuously provide support to students with special needs and their families. After completing this geometry intervention, the investigator wants to use the study results to boost our confidence of other practitioners and educators in delivering educational support at home. Many topics are worth discussing for educators and researchers, such as how to maintain a high level of engagement during online learning, how to provide timely support when asynchronous learning is available, and how to work with school administrators and parents to provide a better home learning environment for students with special needs.

Secondly, geometry knowledge is required at all K-12 levels, and the present geometry intervention extends the literature by filling the gap regarding teaching elementary geometry concepts and skills, such as identifying the attributes of a shape, and determining the line relationships and angle relationships within a 2D geometric shape. The findings of the study suggest that geometry intervention with research-based ICs helped elementary school students with LD acquire elementary geometry skills. The

ICs embedded in the study could be promising for teaching other geometry and mathematics topics. Teachers should have confidence in improving the geometry performance of students with LD and maintaining their skills over time. Therefore, it is encouraged that elementary teachers spend more time providing geometry instruction and support at early grades.

In addition, geometry vocabulary instruction using different instructional strategies was beneficial to vocabulary acquisition. Unlike other mathematics domains, geometry includes more difficult vocabulary, such as parallel lines, perpendicular lines, and acute angles. Many geometry questions with long geometry vocabulary terms become barriers to understanding, and the ability to read geometry questions influences problem-solving (Bay-Williams & Livers, 2009; Powell et al., 2019). The results from the use of the Frayer model and flash cards in this study indicated a positive effect on geometry problem-solving. In addition to the quantitative data collected in this study, the investigator also found the participants had a better mastery of geometry vocabulary. For example, all participants spent less time recognizing the geometry terms during the warming-up and closure sections, and they felt more comfortable in reading and answering the questions. Therefore, additional research is needed to examine the effects of teaching geometry vocabulary on geometry problem-solving skills. It is also recommended that school teachers spend time on geometry vocabulary instruction before introducing problem-solving steps.

Last but not least, it is recommended that elementary teachers collaborate with researchers in various settings, considering that online learning has become a trend in K-

12 education under the new circumstances (Smith & Basham, 2014). Additional work is needed to assess other foundational geometry topics in elementary geometry, such as coordinate planes and shape transformation based on the CCSSM (2010). It is important to consider the students' academic-content needs and individual learning style based on the students' cognitive-development level and create more flexible geometry programs that fit both in-person teaching and online learning with the help of educational technology.

SUMMARY OF THE CHAPTER

The purpose of this study was to examine the effects of geometry intervention on the geometry concepts and skills across three elementary participants with LD using a multiple probe baseline design. The study lasted for about 10 weeks, including the screening and experimental sessions (baseline, intervention, maintenance, and generalization phases). Both a proximal measure (fourth-grade adapted easyCBM geometry) and a distal measure (Keymath-3 geometry subtest) were used to determine the immediate, maintenance, and generalization effects. All participants also completed a social validity form after the intervention.

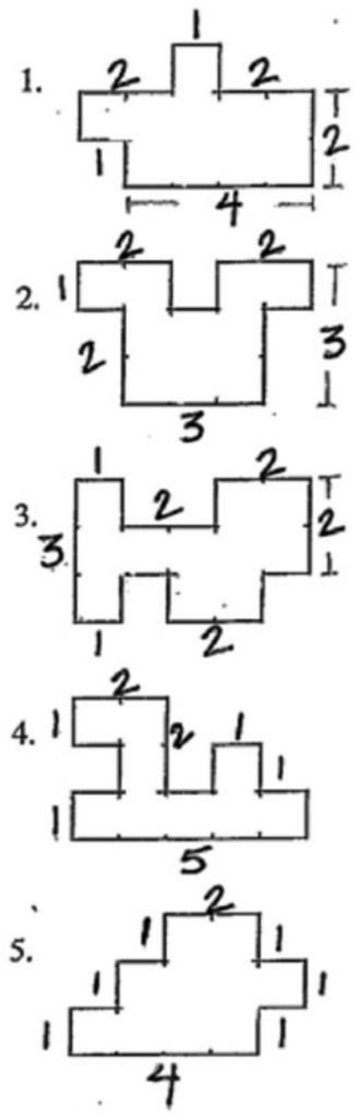
Overall, the participants improved their geometry problem-solving accuracy rate after receiving geometry intervention, which is consistent with previous research (Cass et al., 2003; Cihak & Bowlin, 2009; Horner, 1984; Kozulin & Kazaz, 2016; Satsangi & Bouck, 2015; Satsangi, Hammer, & Bouck, 2019; Satsangi, Hammer, & Hogan, 2019; Strickland & Maccini, 2012; Xin & Hord, 2013). During the baseline phase, all participants displayed stable responses before entering the intervention phase. An immediate intervention effect was observed once the participants had entered the intervention phase. However, because of the interruption of the geometry intervention, the study was paused for a month and had to resume using an online teaching format. Upon the completion of the intervention, two participants reached a relatively high level of performance, while one participant displayed a negative trend due to multiple reasons; for example, the crowded and noisy home environment, and a low level of engagement in online learning. During the maintenance phase, all participants retained a high level of accuracy in their geometry performance. The results of the generalization probes indicated that participants were able to transfer the skills they had learned from the intervention to other contexts. All participants expressed that they had more geometry knowledge after the intervention and could solve more problems. They also enjoyed the geometry vocabulary instructions, which helped them do better in geometry.

Despite several limitations, the findings of the study have provided promising evidence and added to the limited literature by investigating geometry interventions for elementary students with LD. Suggestions for future research and important implications for teachers have been provided; for example, future research on geometry should continue investigating effective strategies and programs to help K-12 students with LD, especially at the elementary level. Teachers are also encouraged to collaborate with researchers and develop programs that work both in person and online.

Appendices

APPENDIX A

Geometry Intervention Irregular Shapes



(Satsangi & Bouck, 2015)

APPENDIX B

Geometry Lesson Sample

Lesson 1: Parallel Lines and Perpendicular Lines

Lesson objectives	Students describe parallel lines and perpendicular lines in 2-dimensional (2d) figures. Students compare and contrast the shape properties of different figures (triangles, rectangles, and squares).	
Vocabulary	points, sides, vertex, vertices, lines, line segments, parallel lines, perpendicular lines, pairs of parallel lines	
Requisite skills	Basic concepts of points and lines	
Misconceptions	Students may not recognize parallel lines and perpendicular lines in a 2d figure.	
Instructional materials	Teacher: AngLegs pattern blocks geoboards practice sheet	Student: Pencils and paper Student notebook

Today we are going to learn about the parallel lines and perpendicular lines. Before we start, we will do some activities to review some of the key terms.

Warming Up (5 min)

1. Review the definition of points and types of lines.

A B

Look at this card. What is this? What type of line is it? (line segment). **It is called line segment. The line segment connects two end points, A and B. The word "segment" is important, because a line normally extends in both directions. But a line segment has a definite length.**

Let's play a game, you will be Point C, and I will be Point D. Let's find a location in this room and decide where C and D are. After we find two locations, ask the question. **If we connect Point C and Point D, what type of line is CD, line or line segment? Why?**

When we talk about the shapes, what type of line are we talking about, lines or line segments? (lines segment with a definite length).

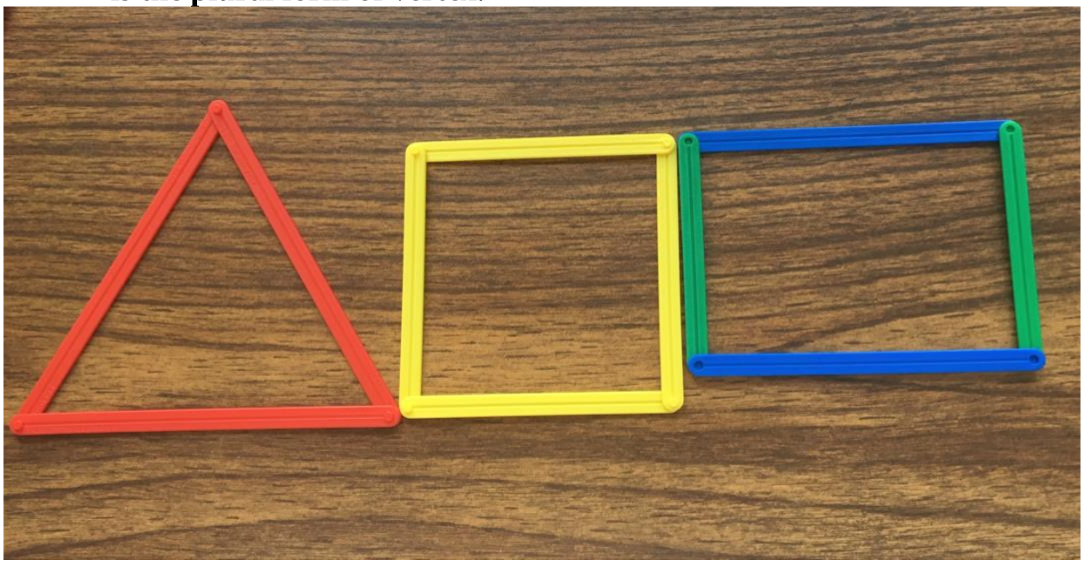
What is a point? A point represents a location in space. Does a triangle have line segments? How many line segments does a triangle have? What are they?

Interactive Modeling (15 min)

1. Vertices and sides

   Look at this Triangle (using AngLegs). Point to one vertex. **This point is called vertex. Read after me, vertex.** Point to other vertices of the triangle. **Is this a vertex, too?**
   This is called the side of a triangle. It is a line segment, because it has a definite length. We use two letters for each of the end points and a line over the top of the letters to represent a side. For example, $\overline{AB}$

   Can you tell me how many vertices/sides does this triangle have? Vertices is the plural form of vertex.



2. A square (using AngLegs)
   Find out the number of: a. Vertices, b. sides.

   Parallel lines are always the same distance apart but will never meet. Let's use our arms to represent parallel lines. Use gesture to represent. **Now look at this square, are the opposite lines parallel?**

   Perpendicular lines are lines that intersects with right angle (90 degree). Let's use our arms to represent perpendicular lines. Use gesture to represent. **How many pairs of parallel lines/Perpendicular lines?** Use the Geocard_L1 to practice answering the questions.

3. A rectangle (AngLegs)
   Ask the students to find vertices, # of sides, parallel lines, perpendicular lines using the Geocard_L1. Find the real-life examples of the parallel lines.

Geocard_L1

Choose a shape, answer:

1. Vertices: ( )
2. sides = line segment: ( )
3. parallel lines: ( ) pairs
4. perpendicular lines ( )

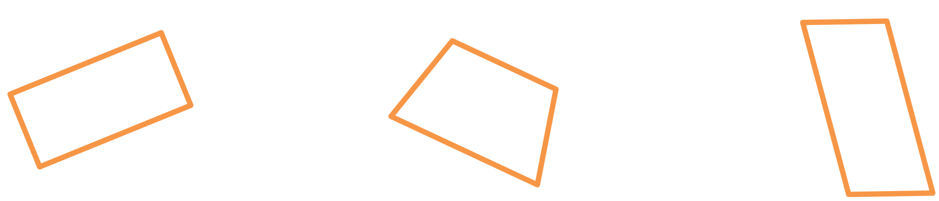
4. **Look at the shape properties of a square and rectangle, what do they have in common? What are their differences?** (The opposite sides have the same length, but the adjacent sides do not.) Consider the items listed on the Geocard_L1.

Guided Practice (10 min)

Let's solve the problems together.

1. **Which line segments are parallel? Point out the parallel lines with your fingers. Are there intersecting lines? Why? Intersecting lines are lines that meet or will meet at some point.**

2. **What do the shapes below have in common? Do they have ___?**
   A. Parallel lines
   B. Perpendicular lines
   C. The same number of sides
   D. The same number of vertices



3. **How many pairs of parallel lines do the shapes in Question 2 have? Label them using a marker.** (2; 1; 2)

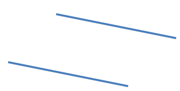
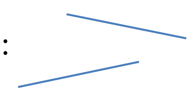
Closure (5 min)

Draw and label the following figures on a piece of paper:

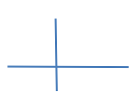
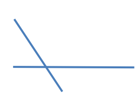
Point A. Point B.
Line segment AB
Draw another line segment that is parallel to line segment AB
Draw a perpendicular line of line AB
Draw a Triangle CDE, and find the vertices and sides

Let's create a journal with Frayer Model for Parallel lines & perpendicular lines.

Parallel lines

Definition: **Parallel lines are always the same distance apart but will never meet.**	Facts: **At least two lines or line segments.**
Examples: 	Non-examples: 

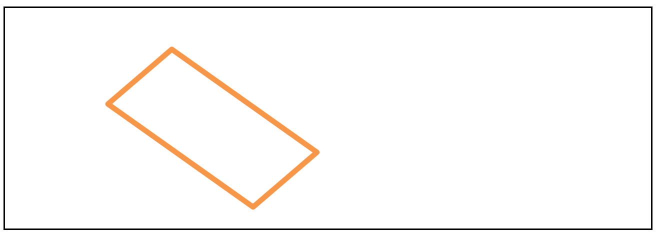
Perpendicular lines

Definition: **Perpendicular lines are lines that intersects with right angle (90 degree).**	Facts: At least two lines. They intersect with 90 degrees.
Examples: 	Non-examples: 

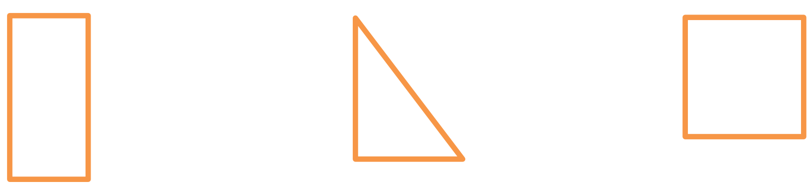
Independent Practice (5 min)

Work on a sheet with 4 problems. Check work and record results. Use error analysis to determine reteaching or extra practice needed. See the example below.

1. How many pairs of parallel lines does the parallelogram have? (write up your answer) ______



2. All the three shapes have _____ (choose all choices that apply, may be more than one answer choice)

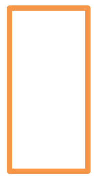


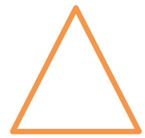
A. Parallel lines
B. Perpendicular lines
C. Acute angles

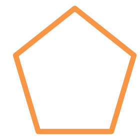
3. Which one shows a single point in space? _____(Choose one answer)

A.
B. .
C.

4. Which figure has perpendicular lines? ______ (Choose one answer)

A. 

B. 

C. 

APPENDIX C

G3 EasyCBM Geometry Sample Form

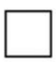
Math Geometry form 1

Student Name:____________________________ Date:__________________

1.

Which is a square?

A. 

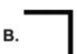
B. 

C. 

2.

Which shape will make this a square?

A. 

B. 

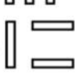
C. 

3.

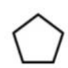
What makes  ?

A. 

B. 

C. 

4.



How many angles inside a pentagon?

A. 5

B. 1

C. 4

5.

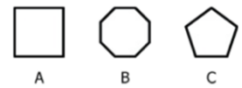
 is $\frac{1}{3}$ of ___

A. 

B. 

C. 

6.



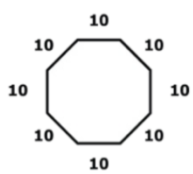
Which has the fewest angles?

A. A

B. B

C. C

7.



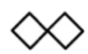
How far is it around this sign?

A. 64

B. 100

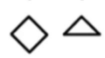
C. 80

8.

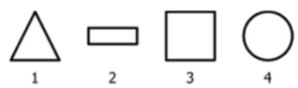
 = ___

A. 

B. 

C. 

9.



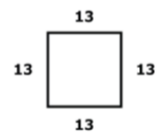
Which is a rectangle?

A. 1

B. 2

C. 4

10.



How far is it around this block?

A. 26

B. 52

C. 48

11.

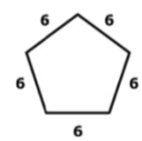
 is $\frac{1}{2}$ of ___

A. 

B. 

C. 

12.



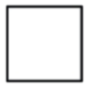
How far is it around this pentagon?

A. 24

B. 36

C. 30

13.



How many ways can you cut this square into 2 equal parts?

A. 2

B. 1

C. 4

14.

Which is a reflection?

A. 

B. 

C. 

15.

What shape can these two triangles make?

A. diamond

B. rectangle

C. circle

16.

Which fits here?

A. 

B. 

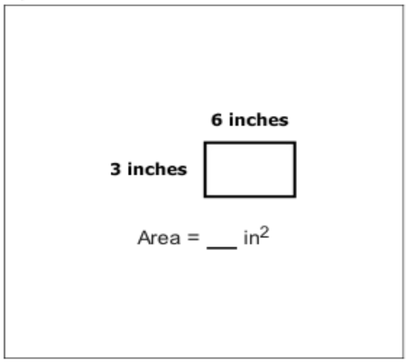
C. 

APPENDIX D

G4 Adapted EasyCBM Geometry Sample Form

Name _______ Date _________

1.

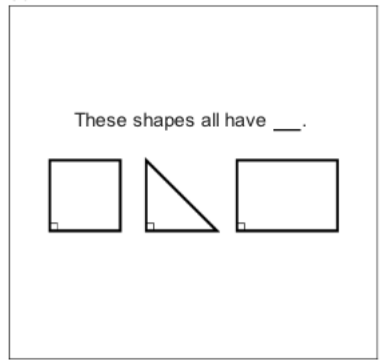


A. 9

B. 24

C. 18

2.

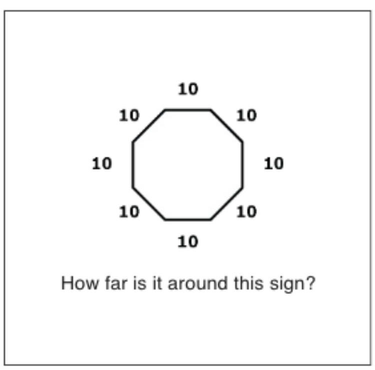


A. acute angles

B. perpendicular lines

C. parallel lines

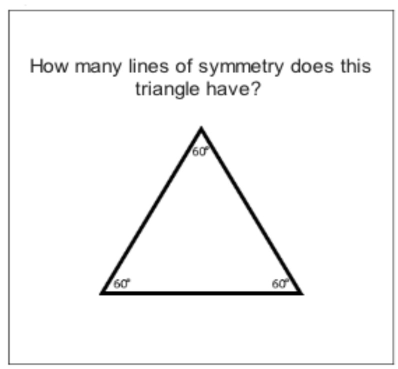
3.



A. 64
B. 100

C. 80

4.



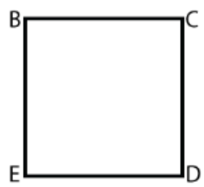
A. 3

B. 2

C. 1

5.

In the square below, which line segments are parallel?



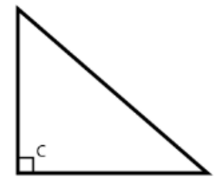
A. BC and BE

B. BE and CD

C. CD and ED

6.

What kind of angle is marked by the letter "C" in this triangle?



A. right

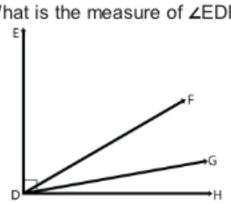
B. obtuse

C. acute

7.

$\angle EDH = 90°$
$\angle FDG = 20°$
$\angle GDH = 10°$

What is the measure of $\angle EDF$?



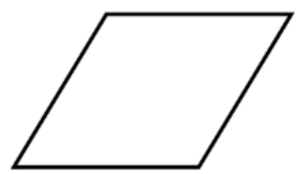
A. 60 degrees

B. 30 degrees

C. 80 degrees

8.

The opposite sides in this parallelogram are ___.



A. perpendicular

B. parallel

C. intersecting

9.

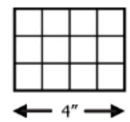
This is a picture of a ___.

A. line segment

B. line

C. ray

10.



Area = ___ in^2

A. 8

B. 10

C. 12

APPENDIX E

Student Social Validity Form: Geometry Intervention

Name: ____________________Date: _________________________________

Directions: Read each statement. Circle your response. Thank you.

1. I like geometry.

Strongly Disagree	Somewhat disagree	Don't agree or disagree	Somewhat agree	Strongly Agree

2. I think geometry is important.

Strongly Disagree	Somewhat disagree	Don't agree or disagree	Somewhat agree	Strongly Agree

3. I know more about shapes and shape attributes to solve geometry problems after the instruction.

Strongly Disagree	Somewhat disagree	Don't agree or disagree	Somewhat agree	Strongly Agree

4. The frayer model help me remembering the knowledge related to shapes.

Strongly Disagree	Somewhat disagree	Don't agree or disagree	Somewhat agree	Strongly Agree

5. The geometry words we learned helped me do better in geometry.

Strongly Disagree	Somewhat disagree	Don't agree or disagree	Somewhat agree	Strongly Agree

6. Using different materials with shapes made geometry easier to understand.

Strongly Disagree	Somewhat disagree	Don't agree or disagree	Somewhat agree	Strongly Agree

7. I feel as though I was able to finish many of the problems independently on the worksheets.

Strongly Disagree	Somewhat disagree	Don't agree or disagree	Somewhat agree	Strongly Agree

8. The flashcards model help me remembering the knowledge related to shapes.

Strongly Disagree	Somewhat disagree	Don't agree or disagree	Somewhat agree	Strongly Agree

9. I think that talking about how to solve a problem helps me understand the problem better.

Strongly Disagree	Somewhat disagree	Don't agree or disagree	Somewhat agree	Strongly Agree

APPENDIX F

Intervention Schedule

February 2020						
Sun	Mon	Tue	Wed	Thu	Fri	Sat
						1
2	3	4	5	6	7	8
9	10	11	12	13 Baseline (Andy, Peggy, & Charlie)	14	15
16	17	18	19 Baseline (A, P, & C)	20	21 Baseline (A, P, & C)	22
23	24 Baseline (A, P, & C)	25	26	27	28 Intervention L1 (A); Baseline (P & C)	29

March 2020

1	2 Intervention L2 (A)	3 Intervention L2 (A)	4 Intervention L3 (A); Baseline (P & C)	5 Intervention L3 (A)	6 Intervention L4 (A); Baseline (P)	7
8	9 Intervention L4 (A)	10 Intervention L5 (A); Intervention L1 (P)	11 Intervention L5 (A); Intervention L2 (P)	12 Intervention L6 (A); Intervention L3 (P); Baseline (C)	13 *Student/staff holiday*	14
15	16 *Student/staff holiday*	17 *Student/staff holiday*	18 *Student/staff holiday*	19 *Student/staff holiday*	20 *Student/staff holiday*	21
22	23 School closed due to COVID-19	24 School closed due to COVID-19	25 School closed due to COVID-19	26 School closed due to COVID-19	27 School closed due to COVID-19	28
29	30 School closed due to COVID-19	31 School closed due to COVID-19				

April 2020

Sun	Mon	Tue	Wed	Thu	Fri	Sat
			1 School closed due to	2 School closed due to	3 School closed due to	4

			COVID-19	COVID-19	COVID-19	
5	6 School closed due to COVID-19	7 School closed due to COVID-19	8 School closed due to COVID-19	9 School closed due to COVID-19	10 School closed due to COVID-19	11
12	13 School closed due to COVID-19	14 School closed due to COVID-19	15 School closed due to COVID-19	16 School closed due to COVID-19	17 School closed due to COVID-19	18
19	20* Intervention L7 (A); Intervention L4 (P); Baseline (C)	21 Intervention L4 (P)	22 Intervention L5 (P); Intervention L1 (C)	23 Intervention L5 (P); Intervention L2 (C)	24 Intervention L6 (P); Intervention L2 (C)	25
26	27 Intervention L6 (P); Intervention L3 (C)	28 Maintenance & generalization (A); Intervention L7 (P); Intervention L3 (C)	29 Intervention L4 (C)	30 Intervention L4 (C)		

May 2020

					1 Intervention L5 (C)	2

3	4 Intervention L5 (C)	5 Maintenance & generalization (P); Intervention L6 (C)	6 Intervention L6 (C)	7 Intervention L7 (C)	8	9
10	11	12	13	14 Maintenance & generalization (C)	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30
31						

Note. * starting from April 20th, 2020, the intervention was online using zoom meetings. A = Andy (in-person sessions: 3:30 pm – 4:00 pm; online sessions 1:00 pm – 1:30 pm); P = Peggy (in-person sessions: 4:00 pm – 4:30 pm; online sessions 11:00 am – 11:30 am); C = Charlie (in-person sessions: 7:30 am to 8:00 am; online sessions 10:00 am – 10:30 am).

APPENDIX G

Fidelity Checklist for Geometry Sample

A. Observation Information

Date: Time:

Lesson: Observer:

B. Content Fidelity

Please check the box (√) for each element based on the observation. 0 represents the element is absent or not observed; 1 represent the element is observed at least once throughout the intervention; NA represent the observer is not sure about whether the element is present or not. If you have questions or concerns, please take notes.

Elements	0	1	NA	Notes
The instructor delivered all the content listed in the lesson objectives.				
The instructor uses a timer to monitor the time spent for each lesson section.				

C. Process Fidelity

Please check the instructional components observed during the session (at least once).

Elements	0	1	NA	Notes
Warming up				
The objectives of the lesson are stated to the students.				
The instructor uses activities to activate student previous knowledge.				
Interactive Modeling				
The instructor scaffolds new concepts and uses modeling when teaching the students.				
The instructor uses questioning strategy to promote student vocabulary and mathematics language.				
The instructor uses different manipulatives, games and activities during teaching (e.g., pictures, pattern blocks).				

Guided Practice				
The instructor provides the students opportunities to practice solve geometry problems.				
The instructor provides ongoing support and feedback to students.				
Closure				
The instructor will help the student to draw a Frayer model on the student notebook.				
The instructor reviews the important items of the lesson.				
Independent Practice				
The student completes geometry problems independently.				
The instructor provides feedback to the students completes the practice.				

D. Suggestions

Instructor's strength: ________________________________

Suggestions for improvement: __________________________________

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